MAT 182: Trigonometry

Final Exam

Spring 2007

Name: Solutions

If you would like your graded exam returned to you please turn in a self addressed stamped (**two** first class stamps) envelope with this exam. Once the exam is graded I will mail it back to you. If you forgot the envelope drop it off within 24 hours to my mail box at the faculty resource center.

No books, notes, friends, or calculators. Sit in every other seat. You have 2 hours for this exam. Answer the questions in the spaces provided. If you run out of room for an answer, write *see back* and continue on the back of the page. One sheet of scratch paper is included on the last page.
If something is unclear quietly come up and ask me.

Unless indicated, angles are in radians. Answers should be given in radians for angles unless requested in degrees.
Simplify all final answers and write in standard form. Show steps where appropriate. **Circle final answers** — if it's unclear what your final answer is or you have multiple answers, full credit cannot be given.

There are 20 questions for a total of 160 points on 15 pages.
Make sure this exam contains all pages.

This Exam is being given under the guidelines of our institution's
Code of Academic Ethics. You are expected to respect those guidelines.

Total Points Earned: __________ **out of 160 total points**

Exam Score: __________

1. (2 points) Convert 10° to radians.

$$360^\circ = 2\pi \text{ rad}$$

$$10^\circ \left(\frac{2\pi}{360^\circ}\right) = \boxed{\frac{\pi}{18}}$$

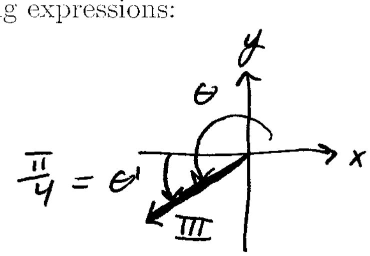
2. Find the exact value of the following expressions:

(a) (5 points) $\csc(5\pi/4)$

$$= \frac{1}{\sin\left(\frac{5\pi}{4}\right)}$$

$$= \frac{1}{-(1/\sqrt{2})}$$

$$\boxed{= -\sqrt{2}}$$



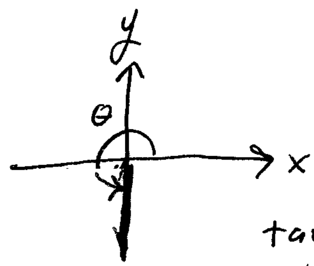
$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Sine is negative in Q III.

(b) (5 points) $\tan(3\pi/2)$

$\therefore$

$$\boxed{= \text{Undefined}}$$

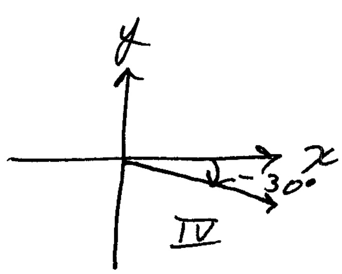


$\tan(\theta) = \frac{y}{x} = \frac{y}{0}$

tangent is undefined!

(c) (5 points) $\cos(-30°)$

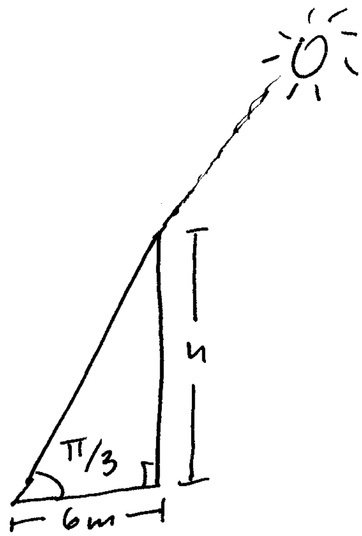
$$\boxed{= \frac{\sqrt{3}}{2}}$$



Cosine is pos in Q IV.

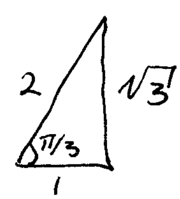
3. A flag pole casts a shadow that measures 6m long. You determine that the sun is at an angle of elevation of $\pi/3$. How high is the flag pole?

   (a) (5 points) Make a simple diagram of this problem labeling the key parts.



   (b) (5 points) Solve for the exact heigh of the flag pole.[1]

$$\tan\left(\pi/3\right) = \frac{h}{6m}$$



$$h = 6m \cdot \tan\left(\pi/3\right)$$

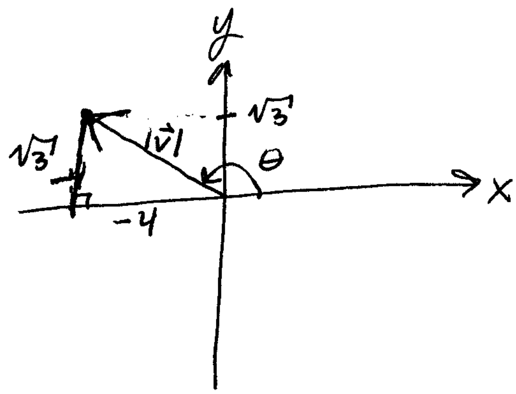
$$= 6m \cdot \left(\frac{\sqrt{3}}{1}\right)$$

$$\boxed{h = 6\sqrt{3}\,m}$$

[1]Don't forget to include units in your answer.

4. (5 points) Let $\vec{v}$ be a vector in standard position pointing in the direction of θ. If its terminal point is $(-4, \sqrt{3})$ find the exact value of $\sin\theta$.

$$\sin(\theta) = \frac{\sqrt{3}}{|\vec{v}|}$$



$$|\vec{v}| = \sqrt{(-4)^2 + (\sqrt{3})^2}$$
$$= \sqrt{16+3}$$
$$= \sqrt{19}$$

$$\sin(\theta) = \frac{\sqrt{3}}{\sqrt{19}}$$

$$\sin(\theta) = \frac{\sqrt{3}\cdot\sqrt{19}}{19}$$

$$\begin{array}{r} \overset{2}{1}9 \\ \times\ 3 \\ \hline 57 \end{array}$$

$$\boxed{\sin(\theta) = \frac{\sqrt{57}}{19}}$$

5. (5 points) List all the values of θ where $\tan\theta$ is undefined on the interval $0 \le \theta \le 2\pi$.

tan θ is undefined at:

$$\boxed{\theta = \pi/2,\ 3\pi/2}$$

for $0 \le \theta \le 2\pi$.

6. (5 points) Prove[2] the following identity:

$$\sec\theta = \cos\theta\,\cot^2\left(\frac{\pi}{2}-\theta\right)+\cos(-\theta)$$

Work with RHS:

$\cos\theta \cot^2\left(\frac{\pi}{2}-\theta\right)+\cos(-\theta)$

$= \cos\theta \cot^2(\pi/2-\theta)+\cos\theta$ [even ID]

$= \cos\theta\left(\cot^2(\pi/2-\theta)+1\right)$ [factor]

$= \cos\theta\left(\tan^2\theta+1\right)$ [cofunction ID]

$\tan^2\theta+1=\sec^2\theta$

$= \cos\theta(\sec^2\theta)$

$= \cancel{\cos\theta}\cdot\frac{1}{\cos^{\cancel{2}}\theta}$

$= \frac{1}{\cos\theta}$

$= \underline{\sec\theta}$ ✓

[2] If your proof is not clear or if it does not follow the conventions we discussed in class you won't get credit for it.

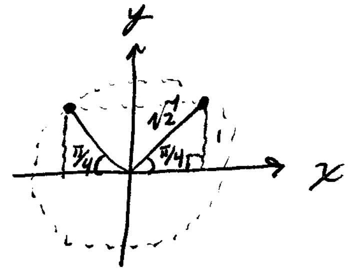
7. (5 points) Find the complete exact solution for the following expressions:

(a) (5 points)

$$\sin 2x = \frac{1}{\sqrt{2}}$$

let $2x = \theta$

On interval $0 \leq \theta < 2\pi$, sine is positive in QI; QII, ∴ 2 solutions.



Then:

$\theta = \pi/4, \frac{3\pi}{4}$ but complete sol is:

$2x = \frac{\pi}{4} + n2\pi, \frac{3\pi}{4} + n2\pi$

$$\boxed{x = \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi}$$

(b) (5 points)

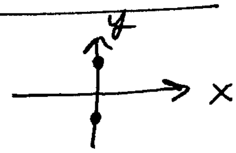
$$\cos x - \sqrt{2}\cos x \, \sin 2x = 0$$

* Factor!

$\cos x \left(1 - \sqrt{2} \sin 2x\right) = 0$

- 2 equations to solve

① $\cos x = 0$



$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad 0 \leq x \leq 2\pi$

$x = \frac{\pi}{2} + n2\pi, \frac{3\pi}{2} + n2\pi$

can combine

$x = \pi/2 + n\pi$

② $(1 - \sqrt{2} \sin 2x) = 0$

$-\sqrt{2} \sin 2x = -1$

$\sin 2x = \frac{1}{\sqrt{2}}$

Answer found in (a)

$x = \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi$

$$\boxed{x = \frac{\pi}{2} + n\pi, \frac{\pi}{8} + n\pi, \frac{3\pi}{8} + n\pi}$$

8. Given the following function

$$f(x) = 3\tan\left(\frac{x}{2}\right)$$

(a) (2 points) The amplitude of $\tan x$ is:

undefined

(b) (2 points) The period of $\tan x$ is:

π

(c) (2 points) The amplitude of $f(x)$ is:

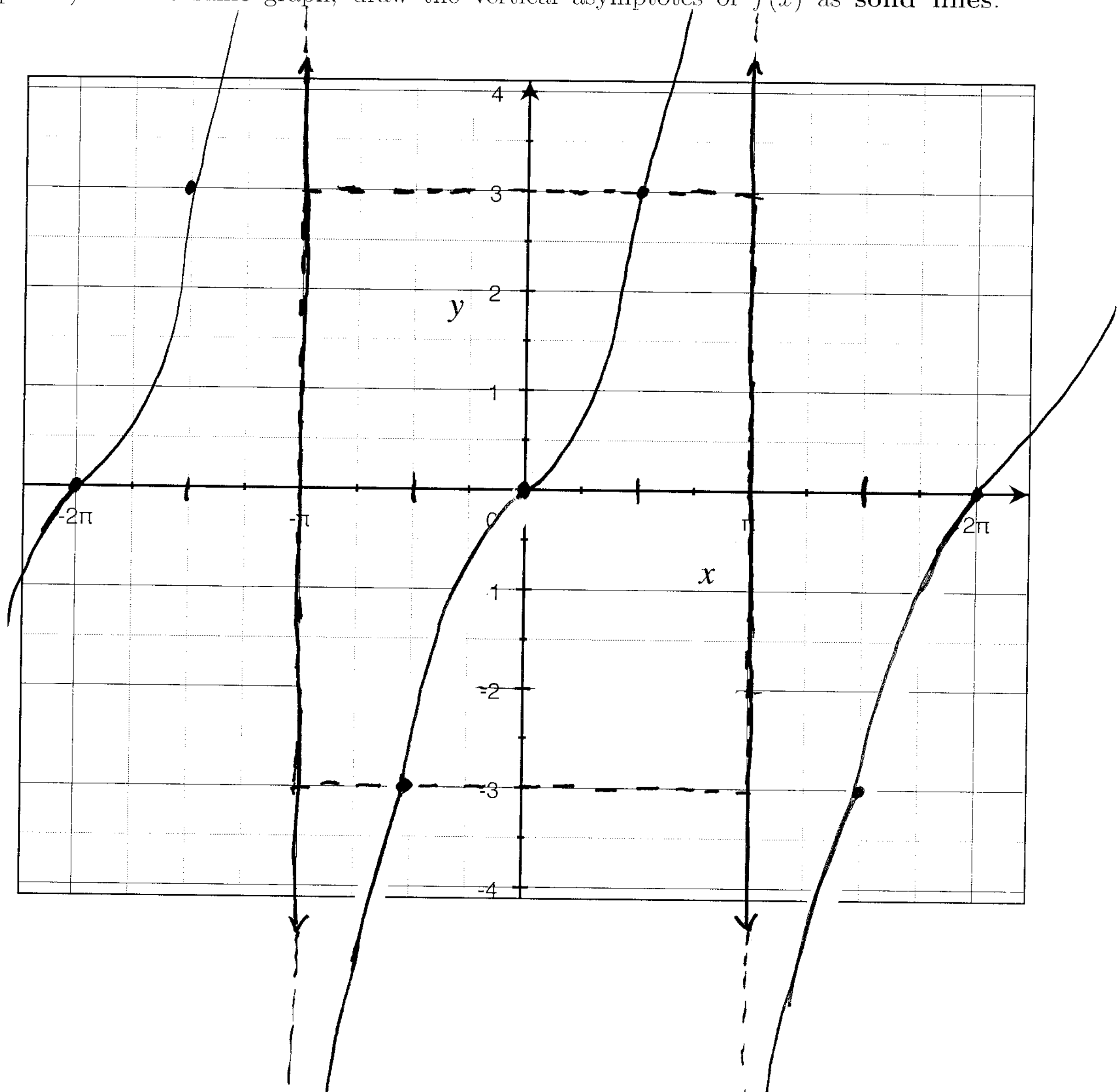
still undefined

(d) (2 points) The period of $f(x)$ is:

$p = \frac{\pi}{b} \quad b = 1/2 \quad \therefore \quad p = 2\pi$

(e) (5 points) Graph $f(x)$ on the interval $[-2\pi, 2\pi]$ on the grid below as a **solid line**.

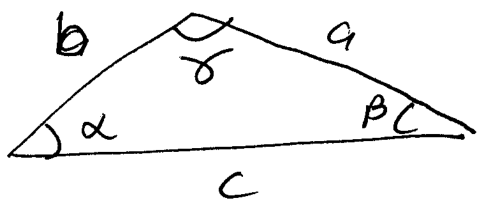
(f) (2 points) On the same graph, draw the vertical asymptotes of $f(x)$ as **solid lines**.



9. (5 points) If you are trying to solve an oblique triangle and you are given two sides and an angle what law should you use?

law of cosines

10. (5 points) Draw an oblique triangle and label the sides and angles.



11. (5 points) Write one valid equation representing the law stated in 9.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

12. (5 points) Can you use the Pythagorean Theorem for *all triangles*?

No, must have a right angle if using Pyth. th.

13. In terms of the vectors $\vec{v}$ and $\vec{w}$:

(a) (5 points) Write an equation used to calculate the angle θ between two vectors $\vec{v}$ and $\vec{w}$.

$$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$$

(b) (5 points) If two vectors are orthogonal what is the angle θ between them?

$\pi/2$ $(90°)$

(c) (5 points) Simplify the equation you wrote in (a) for two orthogonal vectors.

$$\cos\frac{\pi}{2} = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$$

$$0 = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$$

denom. can't be zero ∴

$$\boxed{0 = \vec{v}\cdot\vec{w}}$$

(d) (5 points) If $\vec{v}$ and $\vec{w}$ are defined as

$$\vec{v} = \sqrt{3}\hat{\imath} - \hat{\jmath} \text{ and } \vec{w} = a\hat{\imath} - 2\hat{\jmath}$$

determine the value of a such that $\vec{v}$ and $\vec{w}$ are orthogonal.

From (c) we know

$$0 = \vec{v}\cdot\vec{w}$$

$$0 = v_x w_x + v_y w_y$$

$$0 = \sqrt{3}\cdot a + (-1)(-2)$$

$$0 = a\sqrt{3} + 2$$

$$a = \frac{-2}{\sqrt{3}} \quad \therefore \quad \boxed{a = -\frac{2\sqrt{3}}{3}}$$

14. (2 points) Simplify $(-1)i^4$

$$= (-1) i^2 i^2 = (-1)(-1)(-1)$$

$$\boxed{= -1}$$

15. (5 points) Given the following complex numbers:

$$z_1 = 2i, \text{ and } z_2 = 2 - 4i$$

Simplify and write in standard form:

$$\frac{z_1}{z_2}$$

$$\frac{z_1}{z_2} = \frac{2i}{2-4i} \cdot \frac{(2+4i)}{(2+4i)}$$

$$= \frac{4i + 8i^2}{4 - 16i^2}$$

$$= \frac{-8 + 4i}{4 + 16}$$

$$= \frac{-8 + 4i}{20}$$

$$= -\frac{8}{20} + \frac{4}{20} i$$

$$\boxed{= -\frac{2}{5} + \frac{1}{5} i}$$

16. Given the following polar equation:

$$r^2 \cos^2 \theta = -8r \sin \theta + 8$$

(a) (5 points) Convert the polar equation to cartesian.

$x = r \cos \theta$, $y = r \sin \theta$

$$(\underbrace{r \cos \theta}_{x})^2 = -8 \underbrace{r \sin \theta}_{y} + 8$$

$$x^2 = -8y + 8$$

(b) (5 points) Write your result from (a) in the standard form of a conic section.

$$x^2 = \underbrace{-8}_{4c}(y-1)$$

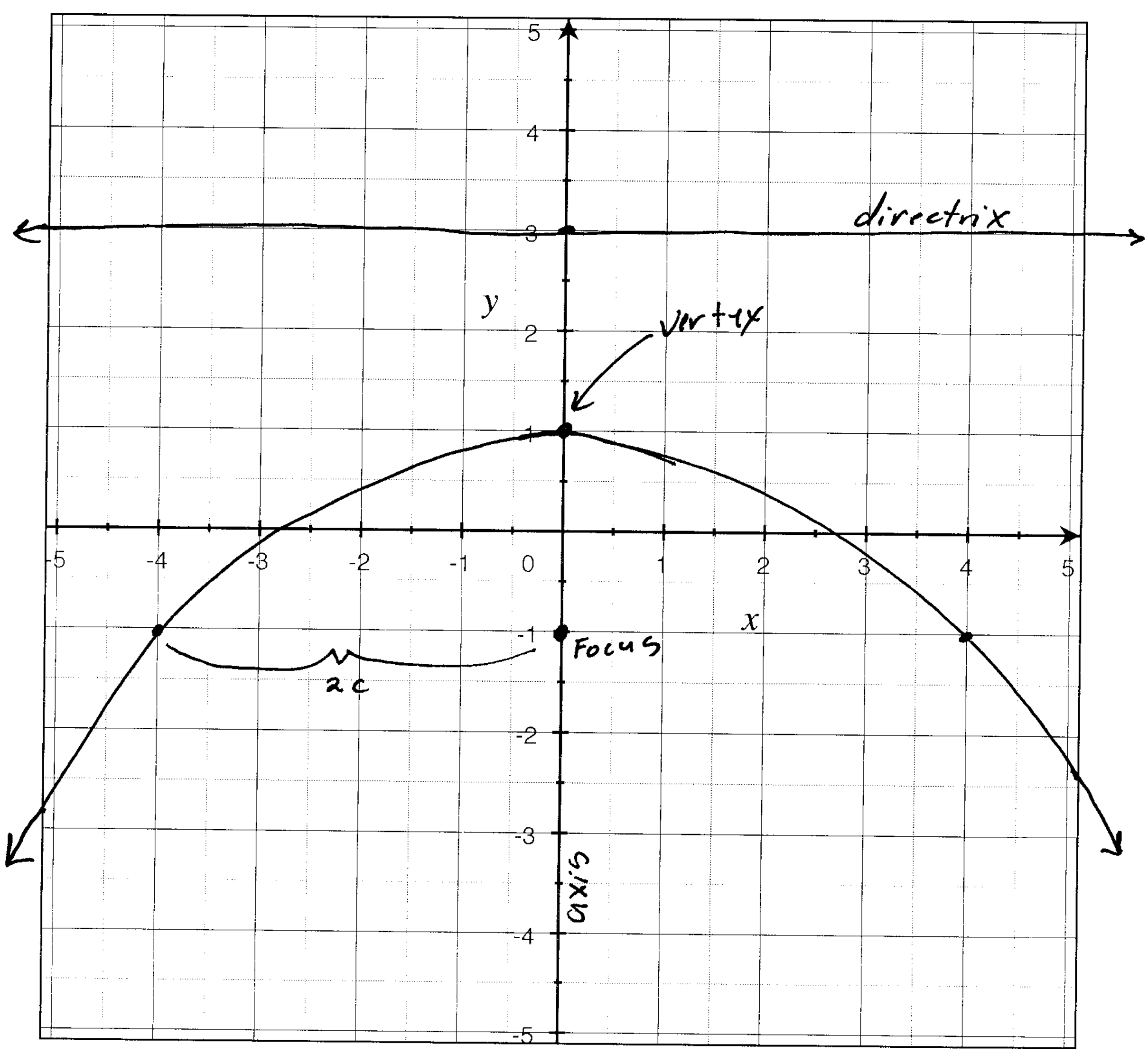
$$x^2 = 4(-2)(y-1)$$

(c) (5 points) Which of the four conic sections does this represent?

Parabola (vertical axis)

(d) (5 points) Make an accurate sketch of the conic section in (b). Ensure all key parts are shown.

$x^2 = 4(-2)(y-1)$



Vertex: $(0, 1)$

Axis: vertical

c: -2

17. (5 points) Which of the four conic sections does the following equation represent?

$$(x-2)^2 - \frac{(y+2)^2}{2} = 4$$

$$\frac{(x-2)^2}{4} - \frac{(y+2)^2}{8} = 1$$

Hyperbola

(Transverse axis horiz)

18. (5 points) Write an equation for an ellipse that has it's major axis in the y direction, a major axis radius of 5, a minor axis radius of 3, and a center at (-2, 0).

$$\frac{(x-(-2))^2}{3^2} + \frac{(y-0)^2}{5^2} = 1$$

$$\boxed{\frac{(x+2)^2}{3^2} + \frac{y^2}{5^2} = 1}$$

19. Given the following polar equation:

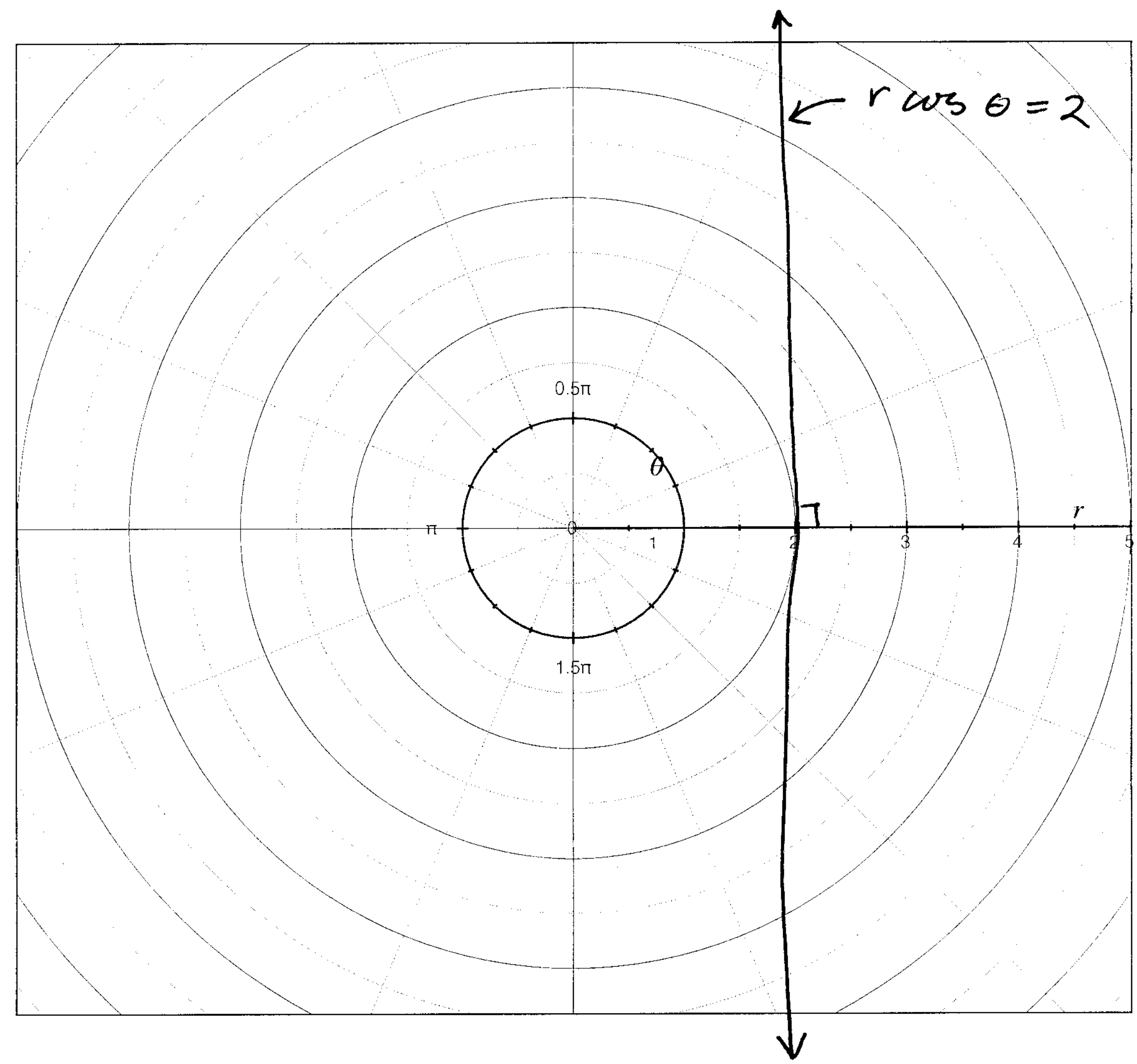
$$r \cos\theta = 2$$

(a) (3 points) What type of graph does this represent?[3]

$r\cos\theta = x \;\therefore\; \underline{x = 2}$

Equation of vertical line

(b) (3 points) Make a graph of the equation on the grid below.

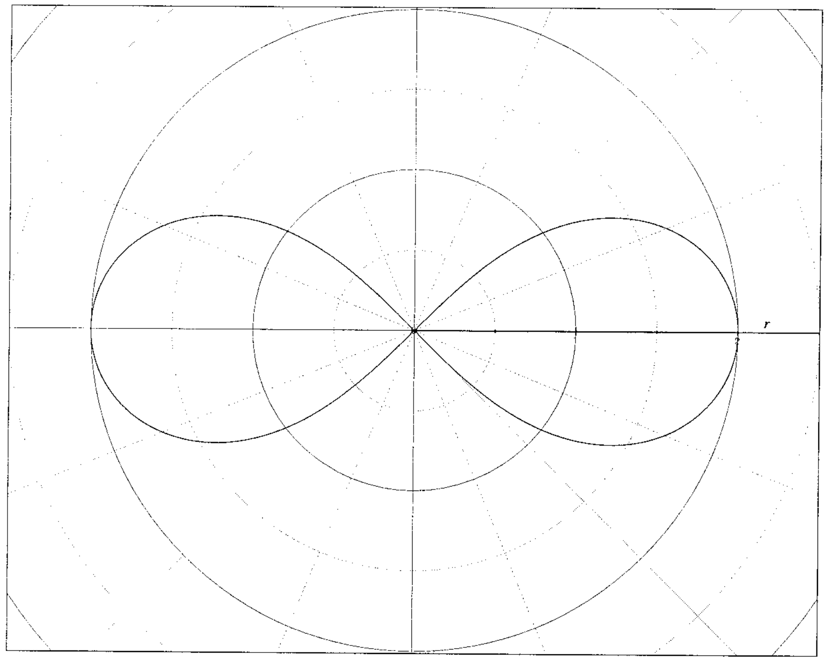


[3]Hint: converting the equation to cartesian may help you to *see* the graph.

20. (5 points) Given the polar function

$$r^2 = 4\cos 2\theta$$

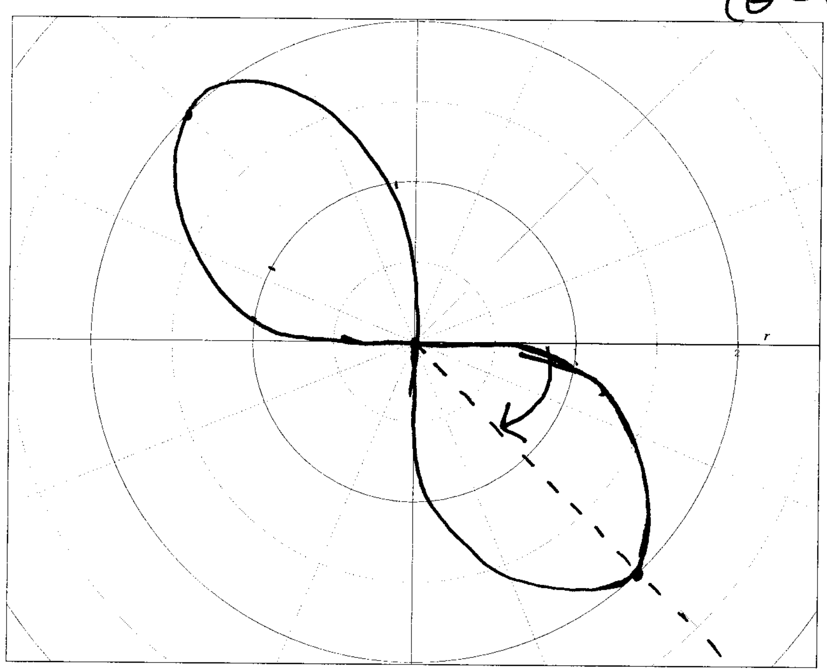
with the graph:



Carefully draw the graph of the following polar function in the grid below:

$$r^2 = 4\cos 2(\theta + \pi/4)$$

θ replaced w/ $(\theta - \alpha)$ rotates by α.
This equation is rotated by $-\pi/4$.



Scratch Paper

Colophon: This exam was typeset using LaTeX with the exam package.