MAT 182: Trigonometry

Exam 3

Spring 2007

Name: Solutions

No books, notes, friends, or calculators. Sit in every other seat. You have 1 hour and 10 minutes for this exam. Answer the questions in the spaces provided. If you run out of room for an answer, write *see back* and continue on the back of the page. One sheet of scratch paper is included on the last page. If something is unclear quietly come up and ask me.

Unless indicated, angles are in radians. Answers should be given in radians for angles unless requested in degrees. Simplify all final answers. Show steps where appropriate. **Circle final answers** — if it's unclear what your final answer is or you have multiple answers, full credit cannot be given.

There are 7 questions for a total of 50 points on 6 pages. Make this exam contains all pages.

This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

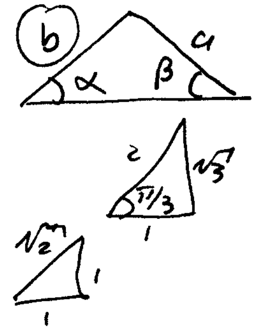
Total Points Earned: ____________ **out of 50 total points**

Exam Score: ____________

1. (5 points) Find the length of the side b given the following parts of an oblique triangle:

$$a = \frac{2}{\sqrt{6}},\ \alpha = \frac{\pi}{4},\ \beta = \frac{\pi}{3}$$

2 angles given ∴ use Law of Sines.



$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$\sin \pi/3 = \frac{\sqrt{3}}{2}$

$\sin \pi/4 = \frac{1}{\sqrt{2}}$

$$b = a\,\frac{\sin\beta}{\sin\alpha}$$

$$= \frac{2}{\sqrt{6}}\,\frac{\sin \pi/3}{\sin \pi/4}$$

$$= \frac{2}{\sqrt{6}}\,\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$$

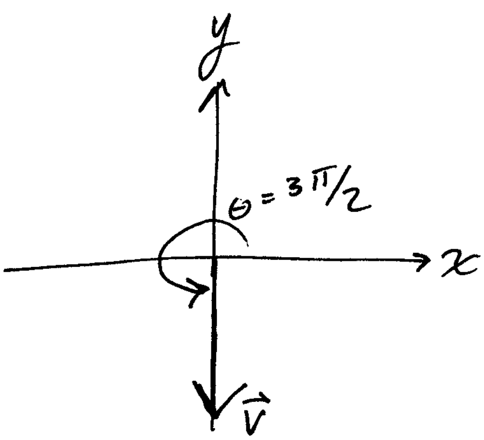
$$b = \frac{2}{\sqrt{6}}\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{1}\right)$$

$$= \frac{2}{\sqrt{6}}\,\frac{\sqrt{6}}{2}$$

$$\boxed{b = 1}$$

2. (5 points) Find the algebraic representation of $\vec{v}$ given:

$$|\vec{v}| = 5,\ \theta = \frac{3\pi}{2}$$



$$\vec{v} = v_x\,\hat{\imath} + v_y\,\hat{\jmath}$$

$$= |\vec{v}|\overset{0}{\cancel{\cos}}\,\theta\cdot\hat{\imath} + |\vec{v}|\sin\theta\cdot\hat{\jmath}$$

$$= 0\,\hat{\imath} - 5\,\hat{\jmath}$$

$$\boxed{= -5\,\hat{\jmath}}$$

Also obvious from sketch.

3. Given the following vectors:

$$\vec{v} = \sqrt{3}\hat{\imath} - \hat{\jmath} \; and \; \vec{w} = -2\hat{\jmath}$$

(a) (5 points) Find $3\vec{v} - \vec{w}$

$$3\vec{v} - \vec{w} = 3(\sqrt{3}\hat{\imath} - \hat{\jmath}) - (-2\hat{\jmath})$$

$$= 3\sqrt{3}\hat{\imath} - 3\hat{\jmath} + 2\hat{\jmath}$$

$$\boxed{= 3\sqrt{3}\hat{\imath} - \hat{\jmath}}$$

(b) (5 points) Find $|\vec{v}|$ and $|\vec{w}|$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{\sqrt{3}^2 + (-1)^2}$$
$$= \sqrt{3+1}$$

$$|\vec{w}| = \sqrt{w_x^2 + w_y^2}$$
$$= \sqrt{0^2 + (-2)^2}$$
$$= \sqrt{4}$$

$$\boxed{|\vec{v}| = 2 \qquad |\vec{w}| = 2}$$

(c) (5 points) Find $\vec{v} \cdot \vec{w}$

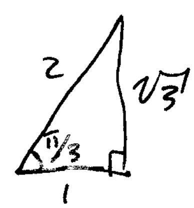
$$\vec{v} \cdot \vec{w} = v_x w_x + v_y w_y$$
$$= \sqrt{3} \cdot 0 + (-1)(-2)$$
$$\boxed{= 2}$$

(d) (5 points) Find the angle θ between $\vec{v}$ and $\vec{w}$

$$\cos\theta = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}|\,|\vec{w}|}$$

$$= \frac{2}{4}$$

$$\cos\theta = \frac{1}{2}$$



solving for θ

$$\boxed{\theta = \pi/3}$$

4. (2 points) Simplify $-i^3$

$$-i^3 = -i^2 \cdot i = -\sqrt{-1}^2 \cdot i = (-1)(-1)i$$
$$\boxed{= i}$$

5. Given the following complex numbers:

$$z_1 = 2 + 3i, \ and \ z_2 = 3 - 4i$$

(a) (5 points) Simplify $(2i)z_1 + z_2$

$$(2i)z_1 + z_2 = 2i(2+3i) + 3 - 4i$$
$$= 4i + 6i^2 + 3 - 4i$$
$$= 4i - 6 + 3 - 4i$$
$$= -3 + 0i$$
$$\boxed{= -3}$$

(b) (5 points) Simplify $\frac{z_1}{z_2}$

$$\frac{z_1}{z_2} = \frac{2+3i}{3-4i}$$ use complex conj of denom. to simplify

$$= \frac{(2+3i)(3+4i)}{(3-4i)(3+4i)}$$
$$= \frac{6+8i+9i+12i^2}{9-16i^2}$$
$$= \frac{6+8i+9i-12}{9+16}$$
$$= \frac{-6+17i}{25}$$
$$\boxed{= \frac{-6}{25} + \frac{17}{25}i}$$

6. (5 points) Convert the following polar equation to rectangular form (write it in terms of x and y.)

$$r = 2\cos\theta$$

$r = 2\cos\theta$

multiply both sides by r

$r^2 = 2r\cos\theta$

$\begin{cases} x = r\cos\theta \\ r^2 = x^2 + y^2 \end{cases}$

$\underline{x^2 + y^2 = 2x}$ ← *ok to stop here

can write in form of circle by completing the square.

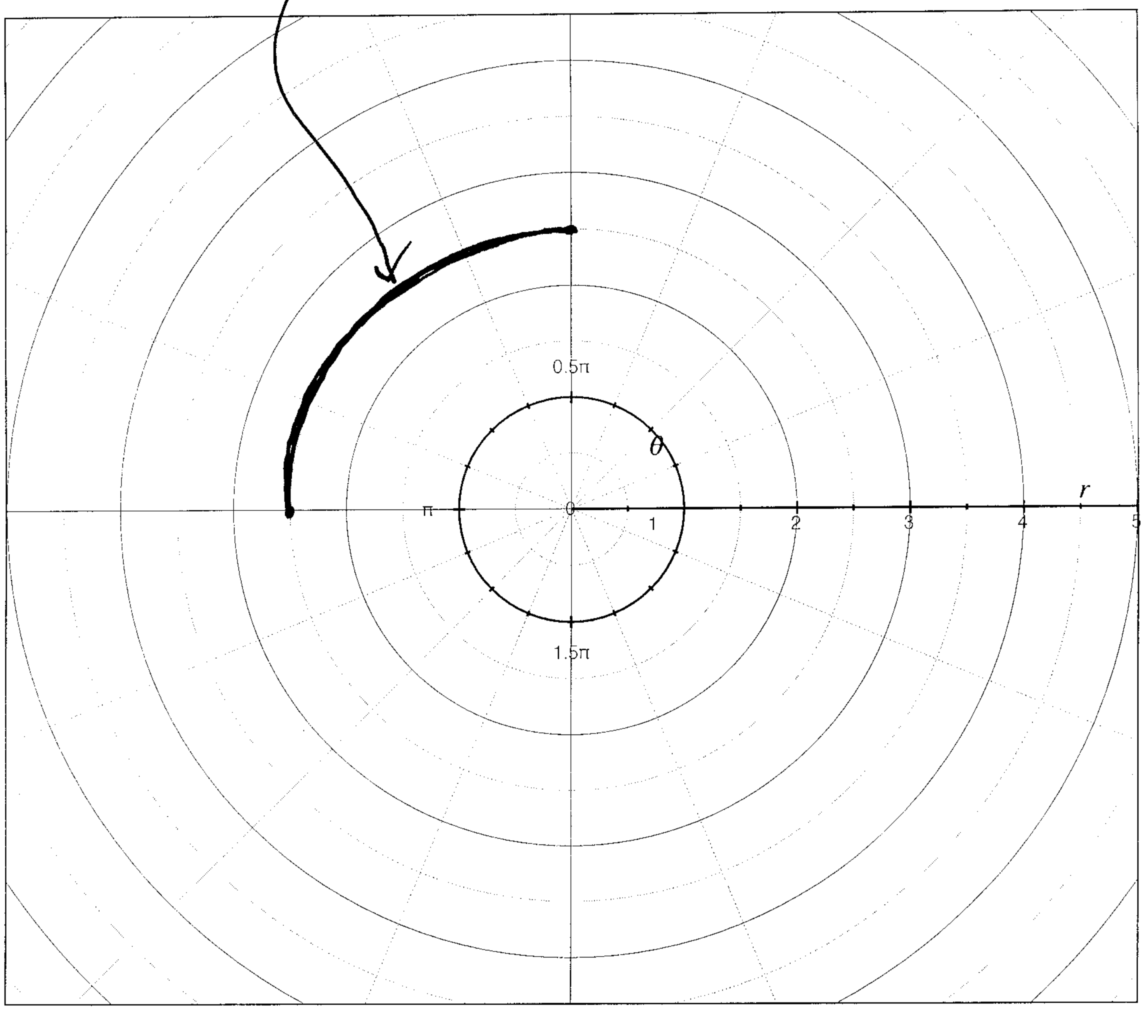
$x^2 - 2x + y^2 = 0$

$(x^2 - 2x + 1) + y^2 = \underline{1}$

$\boxed{(x-1)^2 + y^2 = \underline{1}}$ circle of radius 1 with center at $(1, 0)$

7. (3 points) Plot the following polar equation on the grid below:

$$r = 2.5,\ \frac{\pi}{2} \le \theta \le \pi$$



Scratch Paper

Colophon: This exam was typeset using LaTeX with the exam package.