Introductory Statistics Lectures

Permutations and Combinations

Probability III

ANTHONY TANBAKUCHI
DEPARTMENT OF MATHEMATICS
PIMA COMMUNITY COLLEGE

(Compile date: Tue May 19 14:49:10 2009)

Contents

1 Permutations and Combinations

1.1 Arranging items and factorials

Question 1. If you have 10 items, how many ways can you arrange them? (Think of a tree diagram.)

FACTORIAL NOTATION $x!$. DEFINITION 1.1

Denote product of decreasing whole numbers from x to 1:

$$x! = x \cdot (x-1) \cdot (x-2) \cdot (x-3) \cdots 3 \cdot 2 \cdot 1 \qquad (1)$$

Note: $0! = 1$

FACTORIAL RULE. DEFINITION 1.2

n items can be arranged in $n!$ ways.

R Command

FACTORIAL:
`factorial(x)`

Finds $x!$
(There is a limitation on how large x can be.[a])

[a] The `factorial` function cannot compute values beyond $x \approx 170$ due to how it's implemented using the gamma function. The `lfactorial(x)` function can do larger numbers, it returns $\ln(x!)$.

Example 1. To find 10! in R:

```
R: factorial(10)
[1] 3628800
```

1.2 Permutations and Combinations

PERMUTATIONS

Question 2. How many ways can you select $k = 4$ students out of $n = 10$ when order matters?

Permutations. Definition 1.3

The number of ways (permutations) that you can select k items from n total items (all unique) when **order** matters is:

$$_nP_k = \frac{n!}{(n-k)!} \tag{2}$$

COMBINATIONS

Question 3. How many ways can you select $k = 4$ students out of $n = 10$ when **order does not** matter? (Hint: how many ways can you arrange 4 items?)

DEFINITION 1.4

COMBINATIONS.

The number of combinations (when order does not matter) of k items selected from n different items **without replacement** is:

$$ {}_nC_k = \binom{n}{k} = \frac{n!}{(n-k)!k!} \quad (3) $$

We will need this for binomial probabilities!

R COMMAND

COMBINATIONS:

`choose(n, k)`

Computes number of combinations of k items chosen from a total of n items.

Example 2. To find ${}_{10}C_4$, the number of **combinations** of 4 students chosen from 10:

```
R: choose(10, 4)
[1] 210
```

1.3 Summary

- n items can be arranged in $n!$ ways.
- Number ways you can select k of n items:
  - Permutations: when order matters

$$ {}_nP_k = \frac{n!}{(n-k)!} $$

  - Combinations: when order is unimportant

$$ {}_nC_k = \binom{n}{k} = \frac{n!}{(n-k)!k!} $$

${}_nC_k =$ `choose(n,k)`

1.4 Additional Examples

A Poker hand consists of 5 cards dealt from a deck of 52 cards. (A deck has 2-10, J, Q, K, A — 13 difference valued cards, with 4 suits — 4 of each face.)

Question 4. How many simple events are there in a sample space for a poker hand?

Question 5. A royal flush is a hand that contains A, K, Q, J, 10, all in the same suit. How many ways are there to get a royal flush?

Question 6. What is the probability of being dealt a royal flush?

Question 7. In how many ways can you receive four cards of the same face value and one card from the other 48 available cards?

Question 8. What is the probability of being dealt a 4 of a kind?

Question 9. A businessman in New York is preparing an itinerary for a visit to six major cities. The distance traveled, and hence the cost of the trip, will depend on the order in which he plans his route. How many different itineraries (and trip costs) are possible?

Question 10. In how many ways can you receive three cards of one face value and another two cards of another face value? (A full house)

Question 11. What is the probability of being dealt a full house?