Introductory Statistics Lectures

Multiplication Rule

Probability II

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1 Multiplication Rule

1.1 Motivation

The weather forecast for this week says that there is a 20% chance of rain each day for the work week (Monday - Friday).

Question 1. What is the probability that it will rain on Monday?

Question 2. What is the probability that it will rain on Tuesday?

Question 3. What is the probability that it will rain at least once during the work week?

1.2 Probabilities of multiple trials

Single trial
We previously looked at single trials and learned how to use the addition rule (OR) to calculate probabilities.

Multiple trials
We will now learn how to calculate probabilities for multiple trials using the multiplication rule (AND).

Recall: for a single trial

- Probability of A occurring:

$$P(A) = \frac{x_A}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{total possible number of ways}}$$

- Probability of A or B occurring: (**Never double count**)

$$P(A \text{ or } B) = \frac{x_A}{n} + \frac{x_B}{n}$$

MULTIPLICATION OF CHOICES

Question 4. Make a tree diagram for an experiment that consists of two trials. (1) roll a die and then (2) flip a coin on the second trial. How many possible outcomes are there?

DEFINITION 1.1 MULTIPLICATION OF CHOICES.

If an experiment consists of k trials with n_i possible outcomes for each individual trial, then the total number of **permutations** (outcomes) for the k trials is:

$$N = n_1 \cdot n_2 \cdot n_3 \cdots n_k \tag{1}$$

When order is important. (ie. H-T is different than T-H when flipping a coin.)

Question 5. If an experiment consists of rolling a die 5 times, how many possible outcomes are there?

Question 6. If an experiment consists of (1) rolling a die, (2) flipping a coin, and (3) picking a day of the week, how many possible outcomes are there?

CONDITIONAL PROBABILITY

CONDITIONAL PROBABILITY $P(B|A)$. DEFINITION 1.2

The probability of B occurring given that **A has already occurred**.

Tip for conditional probability
To find $P(B|A)$, just **account for** A **occurring** when finding $P(B)$.

```
R: table(gender, transportation)
          transportation
gender     CAR PUBLIC TRUCK
  FEMALE     9      1     1
  MALE       4      1     2
```

Using the above data for our class find the probability of randomly selecting:

Question 7. A male given that they are a car driver.

Question 8. A car driver given that they are male.

Question 9. A car driver given that they are female.

MULTIPLICATION RULE: AND

Probability of multiple events

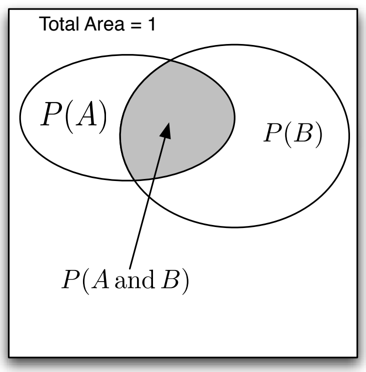
DEFINITION 1.3 MULTIPLICATION RULE: $P(A \text{ AND } B)$.

The probability of A occurring in the first trial **and** B occurring in the second trial.

$$\text{multiplication rule: } P(A \text{ and } B) = P(A) \cdot P(B|A) \quad (2)$$

I'll show you an easy approach in a moment!

Visualizing multiplication rule: Venn diagram



Independent and dependent events

DEFINITION 1.4 INDEPENDENT EVENTS.

B is independent of A if the probability of B occurring is not effected by the occurrence of A.

$$\text{prop. of independence: } P(B|A) = P(B) \quad (3)$$

DEFINITION 1.5 DEPENDENT EVENTS.

The probability of B occurring depends on the occurrence of A.

$$\text{prop. of dependence: } P(B|A) \neq P(B) \quad (4)$$

Can solve for $P(B|A)$ using equation 2.

Determining if events are dependent

If sampling **without** replacement, events are **dependent.**

Independence approximation.

To simplify calculations when sampling without replacement

$$\boxed{\text{treat as independent if: } \frac{n}{N} \leq 0.05} \quad (5)$$

Determine if the following events are independent.

Question 10. Drunk driving and getting a DWI.

Question 11. Getting an A in statistics and getting an A in sculpture.

Question 12. Getting an A on your statistics midterm an getting and A on your statistics final.

Easy approach to the multiplication rule

$$P(A \text{ and } B) = \frac{\text{num. ways A}}{\text{total num. ways}} \cdot \frac{\textbf{num. ways B}}{\textbf{num. ways possible}} \tag{6}$$

- For each successive trial, carefully determine the number of possible successes (numerator) and the total number possible (denominator.)
- If sampling without replacement, both will likely change for each trial.

In our statistics class we have 11 females and 7 males, find the probability of randomly selecting:

Question 13. Two females with replacement.

Question 14. Two females without replacement.

Question 15. Three males without replacement.

Question 16. At a large university, there are 32 males and 660 females in a statistics class, find the probability of selecting 8 males without replacement.

PROBABILITY OF "AT LEAST ONE"

DEFINITION 1.6 PROBABILITY OF "AT LEAST ONE".

To find the probability of "at least one" of something it is **much** easier to find the probability of **none**, then:

$$P(\text{at least one}) = 1 - P(\text{none}) \tag{7}$$

This works because

DEFINITION 1.7 COMPLIMENT OF "AT LEAST ONE".

is **none**.

In our statistics class we have 11 females and 7 males, find the probability of randomly selecting **5 students** with:

Question 17. At least one female with replacement.

Question 18. At least one female without replacement.

1.3 Summary

- Conditional probability $P(B|A)$: pay attention and denominator!
- Independent trials: $P(B|A) = P(B)$
- Treat sampling without replacement as independent when: $\frac{n}{N} \leq 0.05$

- Multiple Trials prob. of A and B: (be careful — count **available**)

$$P(A \text{ and } B) = \frac{\text{num. ways A}}{\text{total num. ways}} \cdot \frac{\textbf{num. ways B}}{\textbf{num. ways possible}}$$

  Be carful when sampling **without replacement**.
- Probability of "At least one": $P(\text{at least one}) = 1 - P(\text{none})$

1.4 Additional examples

The weather forecast for this week says that there is a 20% chance of rain each day for the work week (Monday - Friday).

Question 19. What is the probability that it will rain at least once during the work week?

Question 20. If 4 people are randomly selected, find the probability that no two people were born on the same day of the week.

Question 21. If 4 people are randomly selected, find the probability that at least two people were born on the same day of the week.

Question 22. If couple plans to have 10 children, what is the probability that there will be at least one girl? If the couple eventually has 10 children and they are all boys, what can the couple conclude?

Question 23. In the 108th Congress, the Senate consists of 51 Republicans, 48 Democrats, and 1 Independent. If a lobbyist for the tobacco industry randomly selects three different Senators (without replacement), what is the probability that they are all Republicans?