Introductory Statistics Lectures

Introduction & Addition Rule

Probability I

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1 Introduction & Addition Rule

1.1 Fundamentals

Common Sense

Common sense told us...
Historically, to the observer it was obvious that the sun revolved around the earth via observation.

When to use common sense
Common sense is **dangerous**. Do not use it. Use scientific knowledge based on a body of evidence.

DEFINITIONS

Definition 1.1

Rare event rule.

If we observe an event that has an small probability of being observed, we **conclude that our assumptions about its probability are wrong**. It wasn't random chance. **(Very Important!)**

Example 1. In 1986, the official estimated probability (via modeling) of the shuttle failing was 1/1,000. That year we observed the Challenger Shuttle mission fail.

Definition 1.2

Event: A, B, C.

An outcome or result of a procedure.

Example 2 (Event). Rolling a die (the procedure) and getting a 4 (the event).

Definition 1.3

Compliment of an event: $\bar{A}, \bar{B}, \bar{C}$.

All outcomes in which an event does not occur.

Example 3 (Compliment of an event). Then the compliment of getting a 4 on a die roll is not getting a 4 (or getting a 1, 2, 3, 5, 6).

Definition 1.4

Simple event.

An event that **cannot** be broken down into simpler events.

Definition 1.5

Compound event.

An event that combines two or more simple events.

Example 4 (Compound event). Roll two dice (the procedure) and get sum of 8 (the event).

Example 5 (Simple event). Roll two dice (the procedure) and get a {4, 4}.

Definition 1.6

Sample space.

Set of all possible **simple** events for a procedure.

Example 6 (Sample space). Sample space for rolling one die: $\{1, 2, 3, 4, 5, 6\}$. Has 6 possible events.

Example 7 (Sample space). Sample space for rolling two dice:
$\{\{1, 1\}, \{1, 2\}, \{1, 3\}, \cdots, \{6, 4\}, \{6, 5\}, \{6, 6\}\}$.
Has 36 possible events.

Definition 1.7

Probability of an event: $P(A)$.

The likelihood of an event occurring. Expressed as a number between 0 and 1.

Definition 1.8

Certain event.

$P(A) = 1$

Definition 1.9

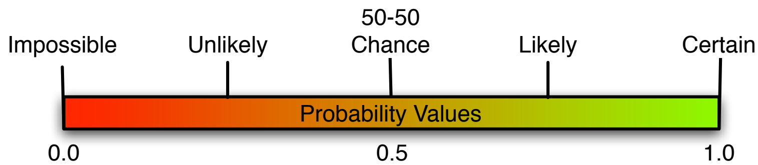
Impossible Event.

$P(A) = 0$

Key property of probabilities

$$0 \leq P(A) \leq 1 \tag{1}$$

Meaning of a probability



Question 1. What is the probability of home work in this class?

Question 2. What is the complement of: home work in this class? What is the probability of this?

Question 3. What is the probability of getting a head on a coin toss?

Question 4. What does it mean if the probability of class ending early is 0.02?

UNUSUAL EVENT. DEFINITION 1.10

An event is unusual if the probability of it occurring is small. In this class, an event is unusual if $P(A) \leq 0.05$. **(Important)**

THREE WAYS TO DEFINE PROBABILITY

1. CLASSICAL APPROACH. DEFINITION 1.11

If a procedure has n different **simple** events, each having an **equal chance** of occurring, then if A occurs x times:

$$\boxed{P(A) = \frac{x}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of events}}} \tag{2}$$

Use when:
- If each simple event has **same chance** of occurring.
- If you can determine n. (Hard for complex problems.)

Example 8 (Classical approach). What is the probability of getting a head on a coin toss?

2. RELATIVE FREQUENCY APPROXIMATION APPROACH. DEFINITION 1.12

Conduct a procedure a large number of times n, the number of occurrences of A (successes) is x, then:

$$P(A) \approx \frac{x}{n} = \frac{\text{number of occurrences of } A}{\text{total number of trials}} \tag{3}$$

Use when:

- Classical approach is too hard (complex problem).
- You can conduct the procedure many times via study, experiment, or simulation.

DEFINITION 1.13 LAW OF LARGE NUMBERS.

As n increases the relative frequency estimate approaches the actual probability.

Using our class as a sample for the population at large, out of 18 students, the modes of transportation are:

```
R: summary(transportation)
   CAR PUBLIC  TRUCK
    13      2      3
```

Question 5. Use the class data to estimate the probability that a randomly chosen person drives a car.

Question 6. Would it be unusual to randomly select a person who drives a car? Why?

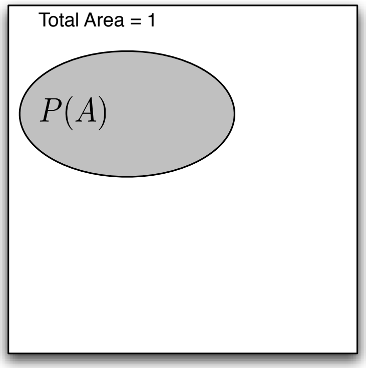
Question 7. How could we increase the accuracy for our estimate of the probability that a person drives a car

DEFINITION 1.14 3. SUBJECTIVE APPROACH (NOT RECOMMENDED).

$P(A)$ is found by estimating its value based on knowledge of the relevant circumstances.

If you can't use the classical approach, then setup a study or an experiment. This method is prone to error and the risks of common sense!

Visualizing probability and sample spaces: Venn diagram



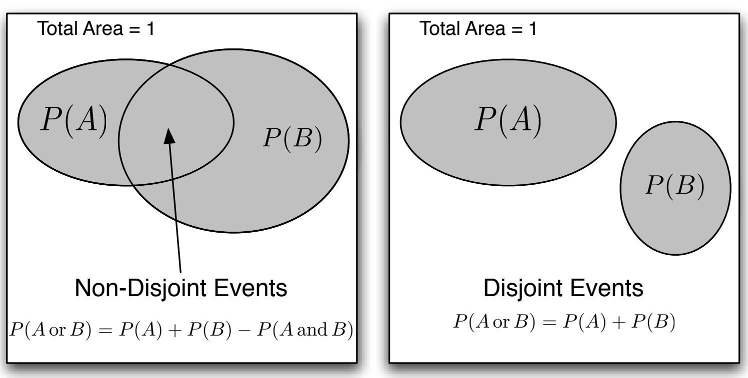
1.2 Probabilities of a single trial

ADDITION RULE: OR

Addition rule. Definition 1.15

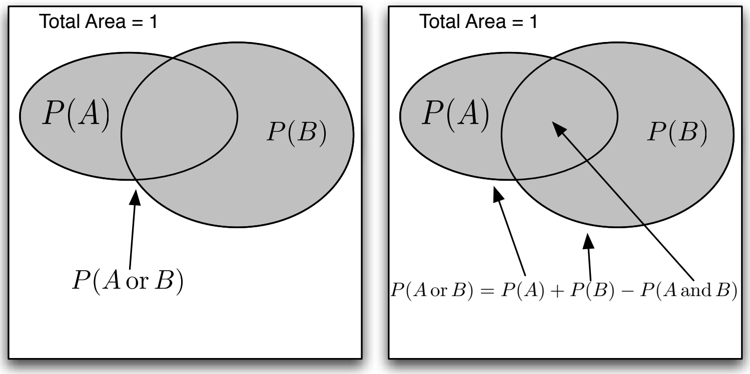
$$\begin{aligned} P(A \text{ or } B) &= P(\text{event } A \text{ or } B \text{ or both occur}) \\ &= P(A) + P(B) - P(A \text{ and } B) \end{aligned}$$

I'll show you an easier method in a moment!



Disjoint events. Definition 1.16

Two events are disjoint (mutually exclusive) if they **cannot** occur together therefore $P(A \text{ and } B) = 0$.



When events are **not** disjoint, we must subtract $P(A \text{ and } B)$ to only count the overlapped area once.

Determine if the following events are disjoint for a single trial:

Question 8. (A) Being Female, (B) Running for President

Question 9. (A) Watching the PBS 6pm news program, (B) Watching the FOX 6pm news program.

Easy method for addition rule

$$\boxed{P(A \text{ or } B) = \frac{\text{number of ways A + B occur}}{\text{total number of ways in S.S.}}} \tag{4}$$

Never double count and this simple method works!!!

```
R: table(gender, transportation)
          transportation
gender     CAR PUBLIC TRUCK
  FEMALE    9      1     1
  MALE      4      1     2
```

Find the probability of randomly selecting (using our class data above):

Question 10. A student that drives a car.

Question 11. A female student.

Question 12. A female student that drives a car.

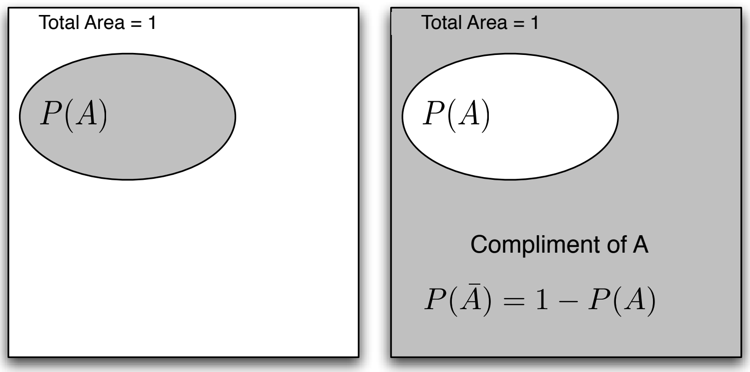
Question 13. A female student or a student who drives a car.

COMPLIMENTARY EVENTS

RULE OF COMPLEMENTARY EVENTS. DEFINITION 1.17

$$P(A) + P(\bar{A}) = 1 \tag{5}$$

Complementary events are **disjoint**.



Question 14. If $P(\bar{A}) = 0.25$ what is the probability of A?

Question 15. If the probability of a car being red is 0.10, what is the probability that a car is not red?

1.3 Odds

ODDS OF SUCCESS. DEFINITION 1.18

$$\boxed{\text{odds of success} = x : f} \tag{6}$$

the ratio of the number of successes x to the **number of failures** f.

ODDS OF FAILURE. DEFINITION 1.19

$$\boxed{\text{odds of failure} = f : x} \tag{7}$$

the ratio of the number of failures f to the number of successes x.

Note that the total number of outcomes n:

$$\boxed{n = x + f} \tag{8}$$

Odds are expressed as integer rations. Customary to give the larger number first. If number of failures is greater than successes, give odds of failure rather than odds of success.

Example 9. The odds of getting a head on a coin toss is 1:1.

Example 10. The odds of not getting a 1 on a die roll is 5:1

In our class of 18 students, the modes of transportation are:

```
R: summary(transportation)
   CAR  PUBLIC   TRUCK
    13       2       3
```

Question 16. What are the odds of randomly selecting a student in this class who drives a car?

Question 17. What are the odds of randomly selecting a student in this class who drives a truck?

Relating probability and odds
If p is the probability of success, with x successes in n trials:

$$p = \frac{x}{n}$$

then $n = x + f$ where f is the number of failures.

Never use odds in calculations, always convert to p.

If the odds against a horse winning a race are 9:1, what is the probability that

Question 18. The horse loses the race?

Question 19. The horse wins the race?

Question 20. If you guess on a multiple choice question and the probability that you get it right is 0.25, what are your odds? (Hint: express p as a fraction, find x, n)

1.4 Summary

- Rare event rule.
- Basic definition of probability:

$$P(A) = \frac{x}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{total possible number of ways}}$$

- Law of large numbers.
- Compliments: $P(\bar{A}) = 1 - P(A)$
- $P(A \, \text{or} \, B)$: (Never double count)

$$P(A \text{ or } B) = \frac{\text{number of ways A + B occur}}{\text{total number of ways}}$$

- Disjoint events cannot occur together.

1.5 Further examples

Basic knowledge for probability questions

- 365 days/year
- 26 letters in the alphabet
- 10 digits $\{0, \ldots, 9\}$
- 52 cards in a deck, 4 suits: $\{1, \ldots 9$, J, Q, K, A$\}$ (9 numbers, 4 faces)

The U.S. General Accounting Office tested the Internal Revenue Service for correctness of answers to taxpayer' questions. In the study, the IRS was correct 1107 times and wrong 626 times.

Question 21. Estimate the probability that a randomly selected taxpayer's question will be answered incorrectly.

Question 22. Is it unusual for the IRS to provide a wrong answer to a taxpayer's question?