Introductory Statistics Lectures

Summation Notation

Compact notation for sums.

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1 Summation Notation

1.1 Introduction

In statistics, we often need to sum sets of numbers.

$$\begin{aligned}&1+2+3+4+5+6+7+8+9+10\\&+11+12+13+14+15+16+17+18+19+20\\&+21+22+23+2+25+26+27+28+29+30\\&+31+32+33+34+35+36+37+38+39+40\\&+41+42+43+44+45+46+47+48+49+50\end{aligned}$$

Writing out a sum can be tedious!
A simpler way to write the above expression would be:

$$\sum_{i=1}^{50} i$$

1.2 Summation notation

SUMMATION NOTATION. DEFINITION 1.1

The summation sign appears as the greek symbol $\sum$ (capitol sigma) and indicates a sequence of sums.

$$\sum_{i=1}^{n} f(i) = \sum_{i=1}^{n} (\text{expression involving } i) \tag{1}$$

$i = 1$ Indicates that the index variable is i and starts at 1.
n The index variable stops at n.
The index variable is always incremented by 1.

Example 1.

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5$$

Question 1. Expand the following expression:

$$\sum_{i=5}^{8} i^2$$

1.3 Notation: Greek alphabet & symbols

Commonly used greek letters in statistics
α, β, ϵ, μ, Σ, σ, Π, ρ, σ, χ

Hat notation for estimates
If we estimate x, we denote it as $\hat{x}$ ("x-hat"). A hat over a variable indicates it is an estimate.

1.4 Notation for sets of data

In statistics we often deal with sets of data. For example, if we have a class of 5 students we can write their ages as:

$$x = \{21, 25, 22, 21, 23\}$$

A	α	Alpha	N	ν	Nu
B	β	Beta	Ξ	ξ	Xi
Γ	γ	Gamma	O	o	Omicron
Δ	δ	Delta	Π	π	Pi
E	ϵ	Epsilon	P	ρ	Rho
Z	ζ	Zeta	Σ	σ	Sigma
H	η	Eta	T	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
I	ι	Iota	Φ	ϕ	Phi
K	κ	Kappa	X	χ	Chi
Λ	λ	Lambda	Ψ	ψ	Psi
M	μ	Mu	Ω	ω	Omega

Table 1: Upper and lower case greek letters.

Where:

$$x_1 = 21$$
$$x_2 = 25$$
$$x_3 = 22$$
$$x_4 = 21$$
$$x_5 = 23$$

Summing a set of data

We can write the sum of the data set $x = \{21, 25, 22, 21, 23\}$ as

$$\sum_{i=1}^{5} x_i = x_1 + x_2 + x_3 + x_4 + x_5$$

If the data set is known, then we can simplify the notation:

$$\sum x_i = 21 + 25 + 22 + 21 + 23$$

Expand the following expressions if $y = \{3, 4, 2, 1\}$

Question 2.

$$\sum (y_i - 1)^2$$

Question 3.

$$\left(\sum(y_i - 1)\right)^2$$

Given $y = \{-a, 3a, a\}$, show the left and right sides are equal by expanding the summation notation and simplifying it. Assume that a is an unknown constant.

Question 4.

$$\sum y_i^2 - 11a = 11a(a-1)$$

1.5 Summation with R

R Command

SUMMATION:
`sum(x)`
Where `x` is a vector.

CAUTION: `sum(x^2)` $= \sum x_i^2$ where `sum(x)^2` $= \left(\sum x_i\right)^2$

Example 2 (Summation in R). Given $x = \{2, 3, 7\}$, find $\sum x_i$:

```
R: x = c(2, 3, 7)

R: total = sum(x)

R: total
[1] 12
```

Example 3 (Summation in R). Now find $\sum(x_i^2 - 2)$:

```
R: sum(x^2 - 2)
[1] 56
```

1.6 Summary

$$\sum_{i=1}^{n}(\text{expression involving } i)$$

$i = 1$ Indicates that the index variable is i and starts at 1.
n The index variable stops at n.

The sum of all the data in x is written as:

$$\sum x_i$$

Summations in R use the `sum(x)` function.