(Compile date: Sat Jan 31 10:52:50 2009).

This homework is probably the hardest formal mathematics that we will do all semester in this course. It is important to become proficient with summation notation so that you can interpret the various equations we will encounter in this class.

Your work must be neat and organized for credit. Show all steps!
The following questions should be done by hand on paper.

1. Show the left and right sides are equal by expanding the summation notation and simplifying it.

(a)

$$\sum_{i=1}^{5} i = 15$$

Solution:

$$\begin{aligned}\sum_{i=1}^{5} i &= 1+2+3+4+5 \\ &= 15\end{aligned}$$

(b)

$$\sum_{i=0}^{3} i^2 = 14$$

Solution:

$$\begin{aligned}\sum_{i=0}^{3} i^2 &= 0^2+1^2+2^2+3^2 \\ &= 14\end{aligned}$$

(c)

$$\sum_{i=0}^{3} (1+i)^2 = 30$$

Solution:

$$\begin{aligned}\sum_{i=0}^{3} (1+i)^2 &= (0+1)^2+(1+1)^2+(1+2)^2+(1+3)^2 \\ &= 30\end{aligned}$$

2. Given $x = \{2, 0, -1, 3\}$, show the left and right sides are equal by expanding the summation notation and simplifying it.

   (a) $\sum x_i = 4$

   Solution:

   $$\begin{aligned}\sum x_i &= 2 + 0 + (-1) + 3 \\ &= 4\end{aligned}$$

   (b) $\sum x_i^2 = 14$

   Solution:

   $$\begin{aligned}\sum x_i^2 &= 2^2 + 0^2 + (-1)^2 + 3^2 \\ &= 4 + 0 + 1 + 9 \\ &= 14\end{aligned}$$

   (c) $(\sum x_i)^2 = 16$

   Solution:

   $$\begin{aligned}\left(\sum x_i\right)^2 &= (2 + 0 + (-1) + 3)^2 \\ &= 4^2 \\ &= 16\end{aligned}$$

   (d) $\sum x_i^2 - 2 = 12$

   Solution:

   $$\begin{aligned}\sum x_i^2 - 2 &= \left[2^2 + 0^2 + (-1)^2 + 3^2\right] - 2 \\ &= [4 + 0 + 1 + 9] - 2 \\ &= 12\end{aligned}$$

   (e) $\sum (x_i - 2)^2 = 14$

   Solution:

   $$\begin{aligned}\sum (x_i - 2)^2 &= (2-2)^2 + (0-2)^2 + (-1-2)^2 + (3-2)^2 \\ &= 0 + 4 + 9 + 1 \\ &= 14\end{aligned}$$

(f) $\sum x_i - \sum x_i^2 = -10$

Solution: Using our previous results:

$$\begin{aligned} \sum x_i - \sum x_i^2 &= 4 - 14 \\ &= -10 \end{aligned}$$

3. Given $y = \{a, -2a, 4a\}$, show the left and right sides are equal by expanding the summation notation and simplifying it. Assume a is an unknown constant.

(a) $\sum y_i = 3a$

Solution:

$$\begin{aligned} \sum y_i &= a + (-2a) + 4a \\ &= 3a \end{aligned}$$

(b) $\sum y_i^2 = 21a^2$

Solution:

$$\begin{aligned} \sum y_i^2 &= a^2 + (-2a)^2 + (4a)^2 \\ &= a^2 + 4a^2 + 16a^2 \\ &= 21a^2 \end{aligned}$$

(c) $(\sum y_i)^2 = 9a^2$

Solution:

$$\begin{aligned} \left(\sum y_i\right)^2 &= (a + (-2a) + 4a)^2 \\ &= (3a)^2 \\ &= 9a^2 \end{aligned}$$

(d) $\sum y_i - a = 2a$

Solution:

$$\begin{aligned} \sum y_i - a &= [a + (-2a) + 4a] - a \\ &= 3a - a \\ &= 2a \end{aligned}$$

(e) $\sum y_i - \sum y_i^2 = 3a(1-7a)$

> **Solution:** Using our previous results:
>
> $$\begin{aligned}\sum y_i - \sum y_i^2 &= 3a - 21a^2 \\ &= 3a(1-7a)\end{aligned}$$

(f) $\frac{\sum y_i - \sum y_i^2}{\sum y_i} = (1-7a)$

> **Solution:** Using our results from the previous questions:
>
> $$\begin{aligned}\frac{\sum y_i - \sum y_i^2}{\sum y_i} &= \frac{3a(1-7a)}{3a} \\ &= 1-7a\end{aligned}$$

4. Rewrite the following R statements in summation notation:

(a) R statement: `sum(z^2)`

> **Solution:**
>
> $$\sum x_i^2$$

(b) R statement: `sum(z)^2`

> **Solution:**
>
> $$\left(\sum x_i\right)^2$$

(c) R statement: `( sum( (z+5)^2 ) + sum(z)^2 ) / 9`

> **Solution:**
>
> $$\frac{\sum (z_i+5)^2 + \left(\sum z_i\right)^2}{9}$$

The following questions should be done using R. Copy your work into a word document. Ensure it is labeled and neat.

5. Define $x = \{2, 0, -1, 3\}$ in R and show the left and right sides are equal by using R to evaluate the left hand side.

   Hint 2: Note that in R: `sum(x^2)` $= \sum x_i^2$ where `sum(x)^2` $= \left(\sum x_i\right)^2$

Solution:

```
> x = c(2, 0, -1, 3)
```

(a) $\sum x_i = 4$

Solution:

```
> sum(x)
[1] 4
```

(b) $\sum x_i^2 = 14$

Solution:

```
> sum(x^2)
[1] 14
```

(c) $\left(\sum x_i\right)^2 = 16$

Solution:

```
> sum(x)^2
[1] 16
```

(d) $\sum x_i^2 - 2 = 12$

Solution:

```
> sum(x^2) - 2
[1] 12
```

(e) $\sum (x_i - 2)^2 = 14$

Solution:

```
> sum((x - 2)^2)
[1] 14
```

(f) $\sum x_i - \sum x_i^2 = -10$

Solution:

```
> sum(x) - sum(x^2)
[1] -10
```

The following questions can be done by hand with a calculator or with R. (Recall that n is the number of data points.)

6. Find the specified measure of center given $x = \{5, 8, 7, 3, 9, 4\}$.
   (a) Find the mean of x.

   Solution:

   ```
   > x = c(5, 8, 7, 3, 9, 4)
   > mean(x)
   [1] 6
   ```

   (Check: 6)

   (b) The harmonic mean is defined as

   $$\text{harmonic mean} = \frac{n}{\sum \frac{1}{x_i}}$$

   find the harmonic mean of x.

   Solution:

   ```
   > n = length(x)
   > n/sum(1/x)
   [1] 5.162171
   ```

   (Check: 5.16)

   (c) The quadratic mean is defined as

   $$\text{quadratic mean} = \sqrt{\frac{\sum x_i^2}{n}}$$

   find the quadratic mean of x.

   Solution:

   ```
   > sqrt(sum(x^2)/n)
   [1] 6.377042
   ```

   (Check: 6.38)

(d) The geometric mean is defined as

$$\text{geometric mean} = \sqrt[n]{\prod x_i}, \quad x_i > 0$$

find the geometric mean of x.
Note that $\prod x_i = x_1 \cdot x_2 \cdot x_3 \cdots x_n$

Solution:

```
> (prod(x))^(1/n)

[1] 5.581663
```

(Check: 5.58)