SOLUTIONS

MAT 167: Statistics

Final Exam

Instructor: Anthony Tanbakuchi

Fall 2008

Name: _____

Computer / Seat Number: _____

No books, notes, or friends. **Show your work.** You may use the attached equation sheet, R, and a calculator. No other materials. If you choose to use R, write what you typed on the test. Using any other program or having any other documents open on the computer will constitute cheating.

You have until the end of class to finish the exam, manage your time wisely.
If something is unclear quietly come up and ask me.
If the question is legitimate I will inform the whole class.

Express all final answers to 3 significant digits. Probabilities should be given as a decimal number unless a percent is requested. Circle final answers, ambiguous or multiple answers will not be accepted. Show steps where appropriate.

The exam consists of 21 questions for a total of 71 points on 14 pages.

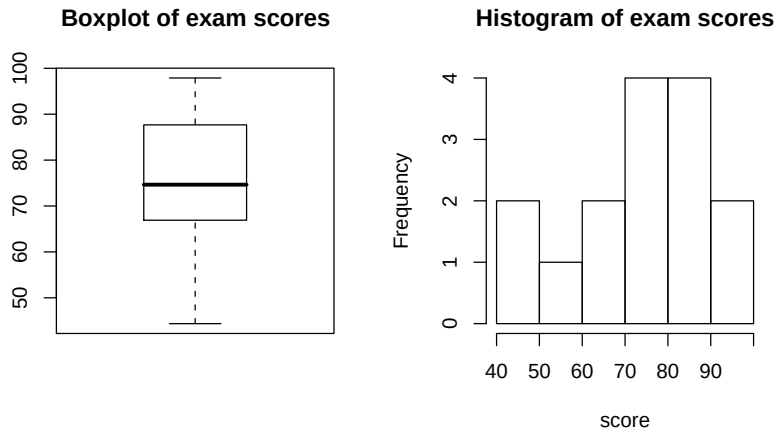
This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

Points Earned: _____ out of 71 total points

Exam Score: _____

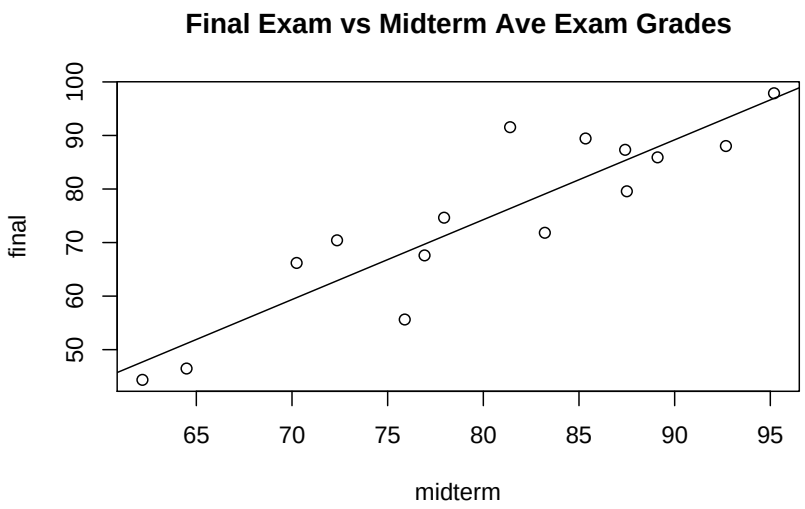
Solution: Exam Results:

```
> summary(score)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  44.37   66.90   74.65   74.46   87.68   97.89
> par(mfrow = c(1, 2))
> boxplot(score, main = "Boxplot of exam scores")
> hist(score, main = "Histogram of exam scores")
```



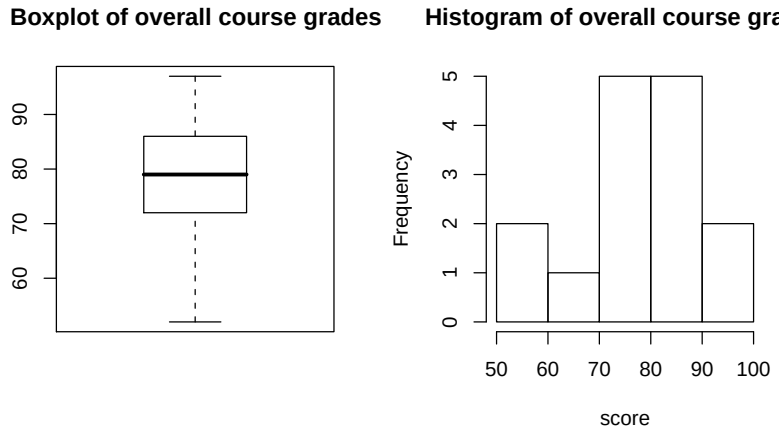
Comparison of midterm average to final exam grades:

```
> plot(midterm, final, main = "Final Exam vs Midterm Ave Exam Grades")
> cor.test(midterm, final)
        Pearson's product-moment correlation

data:  midterm and final
t = 7.4703, df = 13, p-value = 4.695e-06
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.7209078 0.9668203
sample estimates:
      cor
0.9005885
> res = lm(final ~ midterm)
> res
Call:
lm(formula = final ~ midterm)

Coefficients:
(Intercept)      midterm
    -45.048        1.492
> abline(res)
```



Overall Course Grades:

```
> summary(score)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  52.00   72.00   79.00   77.53   86.00   97.00
> par(mfrow = c(1, 2))
> boxplot(score, main = "Boxplot of overall course grades")
> hist(score, main = "Histogram of overall course grades")
```



1. The following is a partial list of statistical methods that we have discussed:

   1. mean
   2. median
   3. mode
   4. standard deviation
   5. z-score
   6. percentile
   7. coefficient of variation
   8. scatter plot

9. histogram
10. pareto chart
11. box plot
12. normal-quantile plot
13. confidence interval for a mean
14. confidence interval for difference in means
15. confidence interval for a proportion
16. confidence interval for difference in proportions
17. one sample mean test
18. two independent sample mean test
19. one sample proportion test
20. two sample proportion test
21. test of homogeneity
22. test of independence
23. linear correlation coefficient & test
24. regression
25. 1-way ANOVA

For each situation below, which method is most applicable?

- If it's a hypothesis test, **also state what the null and alternative hypothesis are**.
- If it's a graphical method, **also describe what you would be looking for**.
- If it's a statistic, how susceptible to outliers is it?

(a) (2 points) A student needs to conduct at t-test on a small set of data. Before conducting the test, the student needs determine if the data appears to have a normal distribution.

Solution: Plot the data with a histogram and see if it looks like a normal distribution. If there are no outliers and it does not appear skewed, then closely analyze it with a Q-Q norm plot. The data should fall close to a line on the Q-Q norm plot if it has a normal distribution.

(b) (2 points) A department of labor researcher wants to determine if there is a dependence between an individual's ethnicity and the type of industry they work in.

Solution: Use the test of independence since both variables are categorical in nature and we are looking for a relationship between them. H_0 : the type of industry a person works in is independent of their ethnicity. H_a : the type of industry a person works in is dependent on their ethnicity.

(c) (2 points) A drug research would like to test the claim that the mean absorption of 1 gram of vitamin E is the same for four methods of delivery: topical, intravenous, oral, and nasal spray.

Solution: Use 1-way ANOVA. H_0 : mean absorption is the same for all four methods. H_a : mean absorption is different for at least one method.

(d) (2 points) A researcher would like to estimate the proportion of people who are dyslexic.

Solution: Conduct a study and construct a confidence interval for a proportion.

(e) (2 points) A fertility researcher wants to determine if a new drug can decrease the proportion of infertile mice. Twenty mice are randomly divided into two groups, a treatment group and a control group.

Solution: Use a two sample hypothesis test of proportions. $H_0 : p_1 = p_2$, $H_a : p_1 < p_2$. (Let group 2 be the control.)

2. (1 point) The test of homogeneity can be thought of as a generalization of what two sample test?

Solution: The two sample proportion test.

3. (1 point) If the mean, median, and mode for a data set are all the same, what can you conclude about the data's distribution?

Solution: If all three measures of center are the same, the distribution is symmetrical. (Not necessarily a normal distribution, all we know is that it is symmetrical.)

4. (2 points) Under what conditions can we approximate a binomial distribution as a normal distribution?

Solution: If the requirements for a binomial distribution are met, it can be approximated as a normal distribution when : $np, nq \geq 5$. In words, there must be at least five successes and failures.

5. (1 point) What percent of data lies within three standard deviations of the mean as stated by the Empirical Rule?

Solution: 99.7%

6. (1 point) Why is it important to use random sampling?

Solution: To prevent bias. Most statistical methods assume random sampling therefore the results will only be reliable if we ensure the assumptions are valid.

7. For the following statements, determine if the calculation requires the use of a **population distribution** or a **sampling distribution**.

   (a) (1 point) Computing a confidence interval for a mean.

Solution: Sampling distribution. We need to utilize the distribution of the sample means.

(b) (1 point) Computing an interval that contain 95% of individual's weights.

Solution: Population distribution. We need to utilize the distribution of individual's weights (the population).

8. (1 point) If the normal approximation to the binomial is valid, write what the following binomial probability statement is approximately equal to in terms of the normal distribution.

$$P_{\text{binom}}(x = 8) \approx$$

Solution: Use the continuity correction.

$$P_{\text{binom}}(x = 8) \approx P_{\text{norm}}(7.5 < x < 8.5)$$

9. (1 point) For the test of homogeneity, what is the distribution of the test statistic? (Give the specific name.)

Solution: χ^2 distribution

10. (2 points) A hypothesis test was conducted for $H_0 : p = 0.5$ and $H_a : p > 0.5$. The test statistic is $z = 1.3$. Find the p-value.

Solution: Since this is a one tailed test. Find the upper tail area on the standard normal distribution.

$$P(z > 1.3) = 1 - F(1.3)$$

```
> p.val = 1 - pnorm(1.3)
> signif(p.val, 3)
[1] 0.0968
```

11. Provide **short succinct** written answers to the following conceptual questions.

(a) (1 point) Would temperature measured in Kelvin be classified as a nominal, ordinal, interval, or ratio level of measurement?

Solution: Ratio since it has a meaningful zero (no energy at 0 K).

(b) (1 point) Which of the following measures of variation is most susceptible to outliers:

standard deviation, inter-quartile range, range

Solution: The range is the least susceptible.

(c) (1 point) What percent of data is greater than Q_2?

Solution: 50%

(d) (1 point) What does the standard deviation represent conceptually **in words**? (Be concise but don't simply state the equation in words verbatim.)

Solution: The standard deviation represents the average variation of the data from the mean.

(e) (2 points) A histogram is a useful tool that can quickly communicate many traits about a set of data. List 4 useful pieces of information that an observer can easily assess using a histogram.

Solution: A histogram can be used to get an approximation of:

1. central tendency
2. variation in the data
3. shape of the data
4. assess if outliers exist
5. min
6. max

(f) (1 point) Why would a SAT percentile be preferred over a raw SAT score for college admissions committees?

Solution: The percentile compares how the applicant did to their peers who took the test (a measure of relative standing). A raw score doesn't give information as to how this score compared to others taking the test, making it hard to determine if a 1100 is easy or hard to get.

12. (2 points) When doing blood testing for HIV infections, the procedure can be made more efficient and less expensive by combining samples of blood specimens. If samples from five people are combined and the mixture tests negative, we know that all five individual samples are negative. Find the probability of a positive result for five samples combined into one mixture, assuming the probability of an individual blood sample testing positive is 10%. (Based on data from the NY State Health Department)

Solution: If multiple individual blood samples are mixed together, the mixture will test

positive if 1 or more individuals are positive. Therefore,

$$\begin{aligned} P(\text{Positive Mixture}) &= P(\text{1 or more samples pos}) \\ &= 1 - P(\text{None positive}) \\ &= 1 - P(\text{NEG \& NEG \& NEG \& NEG \& NEG}) \\ &= 1 - P(NEG)^5 && \text{prop. of independence} \\ &= 1 - (1 - P(POS))^5 && P(NEG) = 1 - P(POS) \\ &= 1 - (1 - 0.1)^5 \\ &= 0.41 \end{aligned}$$

You could also use the binomial distribution:

```
> x = 5
> n = 5
> p = 1 - 0.1
> P = 1 - dbinom(x, n, p)
> signif(P, 3)
[1] 0.41
```

13. (2 points) You would like to conduct a study to estimate (at the 95% confidence level) the proportion of households that own one or more encyclopedias. What sample size do you need to estimate the proportion with a margin of error of 2%.

Solution: Find n using:

$$\text{proportion: } n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2, \qquad (1)$$
$$(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$$

```
> E = 0.02
> alpha = 0.05
> p.hat = 0.5
> q.hat = 0.5
> z.critical = qnorm(1 - alpha/2)
> z.critical
[1] 1.959964
> n = p.hat * q.hat * (z.critical/E)^2
> n
[1] 2400.912
> ceiling(n)
[1] 2401
```

Use a sample size of 2401. (Must round up.)

14. (2 points) If a class consists of 20 males and 8 females, what is the probability of drawing 4 females without replacement?

Solution:

$$P(\text{4 females}) = \frac{8}{28} \cdot \frac{7}{27} \cdot \frac{6}{26} \cdot \frac{5}{25} = 0.00342$$

15. The following questions regard hypothesis testing in general.

(a) (1 point) When we conduct a hypothesis test, we assume something is true and calculate the probability of observing the sample data under this assumption. What do we assume is true?

Solution: We assume the null hypothesis H_0 is true.

(b) (1 point) If you reject H_0 but H_0 is true, what type of error has occurred? (Type I or Type II)

Solution: Type I

(c) (1 point) What variable represents the actual Type I error?

Solution: The p-value. (α is the maximum Type I error, not the actual.)

(d) (1 point) What does the power of a hypothesis test represent?

Solution: The power represents the probability of detecting a true alternative hypothesis.

16. Eighteen students were randomly selected to take the SAT after having either no breakfast or a complete breakfast A researcher would like to test the claim that students who eat breakfast score higher than students who do not.

Group without breakfast: SAT Score	480	510	530	540	550	560	600	620	660
Group with breakfast: SAT Score	460	500	530	520	580	580	560	640	690

(a) (1 point) What type of hypothesis test will you use?

Solution: Use a two sample hypothesis test for equality of means. (The test of independents would not be appropriate since the data is not categorical. Analysis of linear correlation would also be inappropriate since the data is not paired.)

(b) (2 points) What are the test's requirements?

Solution: (1) Simple random samples, (2) the sampling distribution of for both groups is normally distributed (CLT must apply to both samples). (3) Independent samples between groups.

(c) (2 points) What are the hypothesis H_0 and H_a?

Solution: $H_0 : \mu_1 = \mu_2$, $H_a : \mu_1 < \mu_2$ (Let group 1 be the group without breakfast.)

(d) (1 point) What α will you use?

Solution: $\alpha = 0.05$

(e) (2 points) Conduct the hypothesis test. What is the p-value?

Solution:

```
> g1 = c(480, 510, 530, 540, 550, 560, 600, 620, 660)
> g2 = c(460, 500, 530, 520, 580, 580, 560, 640, 690)
> res = t.test(g1, g2, alternative = "less")
> res

        Welch Two Sample t-test

data:  g1 and g2
t = -0.0368, df = 15.239, p-value = 0.4856
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
     -Inf 51.76744
sample estimates:
mean of x mean of y
 561.1111  562.2222
```

The p-value is 0.486.

(f) (1 point) What is your formal decision?

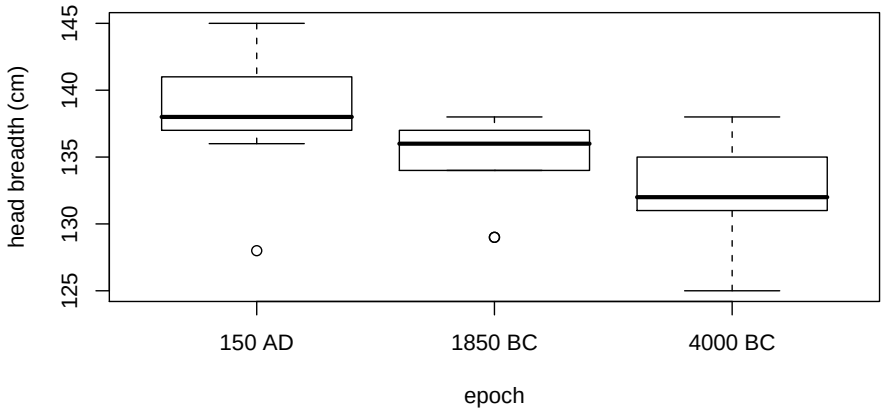
Solution: Since p-val $\not\leq \alpha$, fail to reject H_0.

(g) (2 points) State your final conclusion in words.

Solution: The sample data does not support the claim that students score higher on the SAT when they have breakfast.

17. Samples of head breadths were obtained by measuring skulls of Egyptian males from three different epochs, and the measurements are listed below (based on data from *Ancient Races of the Tebaid*, by Thomas and Randall-Maciver). Changes in head shape over time suggest that interbreeding occurred with immigrant populations. Test the claim that the different epochs do not all have the same mean head breadth.

    A box plot of the data is shown below.



(a) (1 point) What type of hypothesis test (of those discussed in class) should you use?

Solution: 1-Way ANOVA

(b) (1 point) What is the alternative hypothesis for this test?

Solution: H_0 : mean head breadth is different in at least one epoch.

(c) (1 point) What alpha will you use?

Solution: $\alpha = 0.05$

(d) (1 point) What is the response variable for this study?

Solution: head breadth

(e) (1 point) What is the factor variable for this study?

Solution: epoch

(f) (1 point) The analysis of the data was run and the output is shown below: What is your

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
epoch	2	138.74	69.37	4.05	0.0305
Residuals	24	411.11	17.13		

final conclusion (not the formal decision)?

Solution: The sample data supports the claim that different epochs do not all have the same mean head breadth.

(g) (1 point) Assuming the researcher rejected the null hypothesis, what is the probability of a Type I error for this study?

Solution: The p-value = 0.0305

18. The following table lists midterm and final exam grades for eight statistics students.

midterm	75.00	49.00	95.00	82.00	83.00	43.00	95.00	98.00
final	44.00	35.00	82.00	89.00	73.00	43.00	85.00	71.00

(a) (2 points) Upon looking at the scatter plot of the data, the relationship of midterm score and final exam score appears linear. Is the linear relationship statistically significant? (**Justify your answer by conducting an analysis, state the p-value and the conclusion.**)

Solution:

Let x represent the midterm score and y represent the final exam score.

```
> x
[1] 75 49 95 82 83 43 95 98
> y
[1] 44 35 82 89 73 43 85 71
> res = cor.test(x, y)
> res

        Pearson's product-moment correlation

data:  x and y
t = 3.5681, df = 6, p-value = 0.01181
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.2857695 0.9672019
sample estimates:
      cor
0.8244247
```

Yes, there is a statistically significant linear correlation since the p-value ≤ 0.05.

(b) (1 point) What percent of a student's final exam score can be explained by their midterm score?

Solution: $r^2 = 68\%$

(c) (2 points) You want to model a student's final exam score as linearly related to their midterm grade. Write the equation for fitted model (with the actual values of the coefficients). (**Assume the linear correlation is significant.**)

Solution:

```
> res = lm(y ~ x)
> res

Call:
lm(formula = y ~ x)

Coefficients:
(Intercept)            x
     0.3575       0.8373
```

$$\hat{y} = 0.358 + (0.837) \cdot x \quad (2)$$

$$(\text{FINAL}) = 0.358 + (0.837) \cdot (\text{MIDTERM}) \quad (3)$$

(d) (1 point) What range of midterm scores is the model valid for making predictions about final exam scores?

Solution:

```
> range(x)
[1] 43 98
```

(e) (1 point) What is the best predicted final exam score for a student who gets a 75 on the midterm? (**Assume the linear correlation is significant.**)

Solution: Evaluate the above equation for the given score. The best predicted midterm score is 63.2.

(f) (1 point) If the liner relationship had not been statistically significant, what is the best predicted final exam score for a student who gets a 75 on the midterm?

Solution: If the liner correlation is not statistically significant, the best prediction is $\bar{y}$

```
> y.bar = mean(y)
> signif(y.bar, 3)
[1] 65.2
```

19. (2 points) A researcher is trying to determine the ideal temperature to brew coffee. A random sample of 8 of the top 100 coffee shops in New York city had their brewer temperatures (in Celsius) measured. The data is shown below.

88.6, 91.2, 87.5, 90.5, 85.4, 90.6, 93.5, 94.6

Construct a 90% confidence interval for the true population mean temperature using the above data. (**Assume σ is unknown.**)

Solution:

Need to find E in

$$CI = \bar{x} \pm E \tag{4}$$

$$= \bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \tag{5}$$

```
> x
[1] 88.6 91.2 87.5 90.5 85.4 90.6 93.5 94.6
> alpha = 0.1
> n = length(x)
> x.bar = mean(x)
> x.bar
[1] 90.2375
> s = sd(x)
> s
[1] 3.03265
> std.err = s/sqrt(n)
> std.err
[1] 1.072204
> t.crit = qt(1 - alpha/2, df = n - 1)
> t.crit
[1] 1.894579
> E = t.crit * std.err
> E
[1] 2.031374
```

The confidence interval is: 90.2 ± 2.03 or $(88.2, 92.3)$

20. (2 points) A ski resort is designing a new super tram to carry 40 people. If the mean weight of humans is approximately 165 lbs with a standard deviation of 25 lbs, what should the tram's maximum weight limit be so that it can carry the desired capacity 95% of the time?

Solution: The total maximum weight limit = $\bar{x}_{95} \cdot n$. Where $\bar{x}_{95}$ is the sample mean found using the *sampling distribution* of $\bar{x}$ that has an area of 0.95 to the left.

```
> n = 40
> mu = 165
> sigma = 25
> std.err = sigma/sqrt(n)
> std.err
```

```
[1] 3.952847
> x.bar = qnorm(0.95, mean = mu, sd = std.err)
> x.bar
[1] 171.5019
> weight.limit = n * x.bar
> weight.limit
[1] 6860.074
```

21. (2 points) Given $y = \{a, -2a, 4a\}$, completely simplify the following expression. Assume a is an unknown constant.

$$\left(\sum (y_i - 2a)\right)^2$$

Solution:

$$\begin{aligned}\left(\sum (y_i - 2a)\right)^2 &= (a - 2a + -2a - 2a + 4a - 2a)^2 \\ &= (-3a)^2 \\ &= 9a^2\end{aligned}$$

****

End of exam. Reference sheets follow.

Statistics Quick Reference Card & R Commands

by Anthony Tanbakuchi. Version 1.8.2
http://www.tanbakuchi.com
ANTHONY@TANBAKUCHI·COM
Get R at: http://www.r-project.org
R commands: **bold typewriter text**

1 Misc R

To make a vector / store data: **`x=c(x1, x2, ...)`**
Help: general **`RSiteSearch("Search Phrase")`**
Help: function **`?functionName`**
Get column of data from table:
`tableName$columnName`
List all variables: **`ls()`**
Delete all variables: **`rm(list=ls())`**

$$\sqrt{x} = \texttt{sqrt(x)} \quad (1)$$
$$x^n = \texttt{x\^{}n} \quad (2)$$
$$n = \texttt{length(x)} \quad (3)$$
$$T = \texttt{table(x)} \quad (4)$$

2 Descriptive Statistics

2.1 Numerical

Let **`x=c(x1, x2, x3, ...)`**

$$\text{total} = \sum_{i=1}^{n} x_i = \texttt{sum(x)} \quad (5)$$
$$\min = \texttt{min(x)} \quad (6)$$
$$\max = \texttt{max(x)} \quad (7)$$
six number summary : **`summary(x)`** (8)
$$\mu = \frac{\sum x_i}{N} = \texttt{mean(x)} \quad (9)$$
$$\bar{x} = \frac{\sum x_i}{n} = \texttt{mean(x)} \quad (10)$$
$$\tilde{x} = P_{50} = \texttt{median(x)} \quad (11)$$
$$\sigma = \sqrt{\frac{\sum (x_i-\mu)^2}{N}} \quad (12)$$
$$s = \sqrt{\frac{\sum (x_i-\bar{x})^2}{n-1}} = \texttt{sd(x)} \quad (13)$$
$$CV = \frac{\sigma}{\mu} = \frac{s}{\bar{x}} \quad (14)$$

2.2 Relative standing

$$z = \frac{x-\mu}{\sigma} = \frac{x-\bar{x}}{s} \quad (15)$$

Percentiles:

$$P_k = x_i, \quad \text{(sorted } x\text{)}$$
$$k = \frac{i-0.5}{n}\cdot 100\% \quad (16)$$

To find x_i given P_k, i is:

1. $L = (k/100\%)n$
2. if L is an integer: $i = L + 0.5$;
   otherwise i=L and round up.

2.3 Visual

All plots have optional arguments:

- **`main=""`** sets title
- **`xlab="", ylab=""`** sets x/y-axis label
- **`type="p"`** for **p**oint plot
- **`type="l"`** for **l**ine plot
- **`type="b"`** for **b**oth points and lines

Ex: `plot(x, y, type="b", main="My Plot")`
Plot Types:
`hist(x)` histogram
`stem(x)` stem & leaf
`boxplot(x)` box plot
`plot(T)` bar plot, `T=table(x)`
`plot(x,y)` scatter plot, `x, y` are ordered vectors
`plot(t,y)` time series plot, `t, y` are ordered vectors
`curve(expr, xmin,xmax)` plot `expr` involving x

2.4 Assessing Normality

Q-Q plot: **`qqnorm(x); qqline(x)`**

3 Probability

Number of successes x with n possible outcomes.
(Don't double count!)

$$P(A) = \frac{x_A}{n} \quad (17)$$
$$P(\bar{A}) = 1 - P(A) \quad (18)$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (19)$$
$$P(A \text{ or } B) = P(A) + P(B) \quad \text{if } A, B \text{ mut. excl.} \quad (20)$$
$$P(A \text{ and } B) = P(A)\cdot P(B|A) \quad (21)$$
$$P(A \text{ and } B) = P(A)\cdot P(B) \quad \text{if } A, B \text{ independent} \quad (22)$$
$$n! = n(n-1)\cdots 1 = \texttt{factorial(n)} \quad (23)$$
$${}_nP_k = \frac{n!}{(n-k)!} \quad \text{Perm. no elem. alike} \quad (24)$$
$$= \frac{n!}{n_1!n_2!\cdots n_k!} \quad \text{Perm. } n_1 \text{ alike, } \ldots \quad (25)$$
$${}_nC_k = \frac{n!}{(n-k)!k!} = \texttt{choose(n,k)} \quad (26)$$

4 Discrete Random Variables

$$P(x_i): \text{ probability distribution} \quad (27)$$
$$E = \mu = \sum x_i \cdot P(x_i) \quad (28)$$
$$\sigma = \sqrt{\sum (x_i-\mu)^2 \cdot P(x_i)} \quad (29)$$

4.1 Binomial distribution

$$\mu = n\cdot p \quad (30)$$
$$\sigma = \sqrt{n\cdot p\cdot q} \quad (31)$$
$$P(x) = {}_nC_x p^x q^{(n-x)} = \texttt{dbinom(x, n, p)} \quad (32)$$

4.2 Poisson distribution

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \texttt{dpois(x, } \mu\texttt{)} \quad (33)$$

5 Continuous random variables

CDF $F(x)$ gives area to the left of x, $F^{-1}(p)$ expects p is area to the left.

$$f(x): \text{ probability density} \quad (34)$$
$$E = \mu = \int_{-\infty}^{\infty} x\cdot f(x)\,dx \quad (35)$$
$$\sigma = \sqrt{\int_{-\infty}^{\infty} (x-\mu)^2\cdot f(x)\,dx} \quad (36)$$
$$F(x): \text{ cumulative prob. density (CDF)} \quad (37)$$
$$F^{-1}(x): \text{ inv. cumulative prob. density} \quad (38)$$
$$F(x) = \int_{-\infty}^{x} f(x')\,dx' \quad (39)$$
$$p = P(x < x') = F(x') \quad (40)$$
$$x' = F^{-1}(p) \quad (41)$$
$$p = P(x > a) = 1 - F(a) \quad (42)$$
$$p = P(a < x < b) = F(b) - F(a) \quad (43)$$

5.1 Uniform distribution

$$p = P(u < u') = F(u')$$
$$= \texttt{punif(u', min=0, max=1)} \quad (44)$$
$$u' = F^{-1}(p) = \texttt{qunif(p, min=0, max=1)} \quad (45)$$

5.2 Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\cdot e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \quad (46)$$
$$p = P(z < z') = F(z') = \texttt{pnorm(z')} \quad (47)$$
$$z' = F^{-1}(p) = \texttt{qnorm(p)} \quad (48)$$
$$p = P(x < x') = F(x')$$
$$= \texttt{pnorm(x', mean=}\mu\texttt{, sd=}\sigma\texttt{)} \quad (49)$$
$$x' = F^{-1}(p)$$
$$= \texttt{qnorm(p, mean=}\mu\texttt{, sd=}\sigma\texttt{)} \quad (50)$$

5.3 t-distribution

$$p = P(t < t') = F(t') = \texttt{pt(t', df)} \quad (51)$$
$$t' = F^{-1}(p) = \texttt{qt(p, df)} \quad (52)$$

5.4 χ^2-distribution

$$p = P(\chi^2 < \chi^{2'}) = F(\chi^{2'})$$
$$= \texttt{pchisq(}X^{2}\texttt{', df)} \quad (53)$$
$$\chi^{2'} = F^{-1}(p) = \texttt{qchisq(p, df)} \quad (54)$$

5.5 F-distribution

$$p = P(F < F') = F(F')$$
$$= \texttt{pf(F', df1, df2)} \quad (55)$$
$$F' = F^{-1}(p) = \texttt{qf(p, df1, df2)} \quad (56)$$

6 Sampling distributions

$$\mu_{\bar{x}} = \mu \qquad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad (57)$$
$$\mu_{\hat{p}} = p \qquad \sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} \quad (58)$$

7 Estimation

7.1 Confidence Intervals

proportion: $\hat{p} \pm E, \quad E = z_{\alpha/2}\cdot\sigma_{\hat{p}}$ (59)
mean (σ known): $\bar{x} \pm E, \quad E = z_{\alpha/2}\cdot\sigma_{\bar{x}}$ (60)
mean (σ unknown, use s): $\bar{x} \pm E, \quad E = t_{\alpha/2}\cdot\sigma_{\bar{x}}$, (61)
$df = n-1$
variance: $\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$, (62)
$df = n-1$
2 proportions: $\Delta\hat{p} \pm z_{\alpha/2}\cdot\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$ (63)
2 means (indep): $\Delta\bar{x} \pm t_{\alpha/2}\cdot\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$, (64)
$df \approx \min(n_1-1,\, n_2-1)$
matched pairs: $\bar{d} \pm t_{\alpha/2}\cdot\frac{s_d}{\sqrt{n}}, \quad d_i = x_i - y_i$, (65)
$df = n-1$

7.2 CI Critical Values (two sided)

$z_{\alpha/2} = F_z^{-1}(1-\alpha/2) = \texttt{qnorm(1-alpha/2)}$ (66)
$t_{\alpha/2} = F_t^{-1}(1-\alpha/2) = \texttt{qt(1-alpha/2, df)}$ (67)
$\chi^2_L = F_{\chi^2}^{-1}(\alpha/2) = \texttt{qchisq(alpha/2, df)}$ (68)
$\chi^2_R = F_{\chi^2}^{-1}(1-\alpha/2) = \texttt{qchisq(1-alpha/2, df)}$ (69)

7.3 Required sample size

$$\text{proportion: } n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2, \quad (70)$$
$$(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$$
$$\text{mean: } n = \left(\frac{z_{\alpha/2}\cdot\hat{\sigma}}{E}\right)^2 \quad (71)$$

8 Hypothesis Tests

Test statistic and R function (when available) are listed for each.

Optional arguments for **hypothesis tests**:

`alternative="two.sided"` can be:
`"two.sided"`, **`"less"`**, **`"greater"`**

`conf.level=0.95` constructs a 95% confidence interval. Standard CI only when `alternative="two.sided"`.

Optional arguments for **power calculations & Type II error**:

`alternative="two.sided"` can be:
`"two.sided"` or **`"one.sided"`**

`sig.level=0.05` sets the significance level α.

8.1 1-SAMPLE PROPORTION

$H_0 : p = p_0$

`prop.test(x, n, p=`p_0**`, alternative="two.sided")`**

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} \tag{72}$$

8.2 1-SAMPLE MEAN (σ KNOWN)

$H_0 : \mu = \mu_0$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \tag{73}$$

8.3 1-SAMPLE MEAN (σ UNKNOWN)

$H_0 : \mu = \mu_0$

`t.test(x, mu=`μ_0**`, alternative="two.sided")`**

Where **`x`** is a vector of sample data.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, \quad df = n - 1 \tag{74}$$

Required Sample size:

`power.t.test(delta=`h**`, sd =`**σ**`, sig.level=`**α**`, power=`**$1 - \beta$**`, type ="one.sample", alternative="two.sided")`**

8.4 2-SAMPLE PROPORTION TEST

$H_0 : p_1 = p_2$ or equivalently $H_0 : \Delta p = 0$

`prop.test(x, n, alternative="two.sided")`

where: **`x=c(`**x_1**`, `**x_2**`)`** and **`n=c(`**n_1**`, `**n_2**`)`**

$$z = \frac{\Delta\hat{p} - \Delta p_0}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}, \quad \Delta\hat{p} = \hat{p}_1 - \hat{p}_2 \tag{75}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}, \quad \bar{q} = 1 - \bar{p} \tag{76}$$

Required Sample size:

`power.prop.test(p1=`p_1**`, p2=`**p_2**`, power=`**$1 - \beta$**`, sig.level=`**α**`, alternative="two.sided")`**

8.5 2-SAMPLE MEAN TEST

$H_0 : \mu_1 = \mu_2$ or equivalently $H_0 : \Delta\mu = 0$

`t.test(x1, x2, alternative="two.sided")`

where: **`x1`** and **`x2`** are vectors of sample 1 and sample 2 data.

$$t = \frac{\Delta\bar{x} - \Delta\mu_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \quad df \approx \min(n_1 - 1, n_2 - 1), \quad \Delta\bar{x} = \bar{x}_1 - \bar{x}_2 \tag{77}$$

Required Sample size:

`power.t.test(delta=`h**`, sd =`**σ**`, sig.level=`**α**`, power=`**$1 - \beta$**`, type ="two.sample", alternative="two.sided")`**

8.6 2-SAMPLE MATCHED PAIRS TEST

$H_0 : \mu_d = 0$

`t.test(x, y, paired=TRUE, alternative="two.sided")`

where: **`x`** and **`y`** are ordered vectors of sample 1 and sample 2 data.

$$t = \frac{\bar{d} - \mu_{d0}}{s_d/\sqrt{n}}, \quad d_i = x_i - y_i, \quad df = n - 1 \tag{78}$$

Required Sample size:

`power.t.test(delta=`h**`, sd =`**σ**`, sig.level=`**α**`, power=`**$1 - \beta$**`, type ="paired", alternative="two.sided")`**

8.7 TEST OF HOMOGENEITY, TEST OF INDEPENDENCE

$H_0 : p_1 = p_2 = \cdots = p_n$ (homogeneity)

$H_0 : X$ and Y are independent (independence)

`chisq.test(D)`

Enter table: **`D=data.frame(c1, c2, ...)`**, where `c1, c2, ...` are column data vectors.

Or generate table: **`D=table(x1, x2)`**, where `x1, x2` are ordered vectors of raw categorical data.

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}, \quad df = (\text{num rows - 1})(\text{num cols - 1}) \tag{79}$$

$$E_i = \frac{(\text{row total})(\text{column total})}{(\text{grand total})} = np_i \tag{80}$$

For 2×2 contingency tables, you can use the Fisher Exact Test:

`fisher.test(D, alternative="greater")`

(must specify alternative as greater)

9 Linear Regression

9.1 LINEAR CORRELATION

$H_0 : \rho = 0$

`cor.test(x, y)`

where: **`x`** and **`y`** are ordered vectors.

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}, \quad t = \frac{r - 0}{\sqrt{\frac{1-r^2}{n-2}}}, \quad df = n - 2 \tag{81}$$

9.2 MODELS IN R

MODEL TYPE	EQUATION	R MODEL
linear 1 indep var	$y = b_0 + b_1 x_1$	`y~x1`
...0 intercept	$y = 0 + b_1 x_1$	`y~0+x1`
linear 2 indep vars	$y = b_0 + b_1 x_1 + b_2 x_2$	`y~x1+x2`
...inteaction	$y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2$	`y~x1+x2+x1*x2`
polynomial	$y = b_0 + b_1 x_1 + b_2 x_2^2$	`y~x1+I(x2^2)`

9.3 REGRESSION

Simple linear regression steps:

1. Make sure there is a significant linear correlation.
2. **`results=lm(y~x)`** Linear regression of `y` on x vectors
3. **`results`** View the results
4. **`plot(x, y); abline(results)`** Plot regression line on data
5. **`plot(x, results$residuals)`** Plot residuals

$$y = b_0 + b_1 x_1 \tag{82}$$

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2} \tag{83}$$

$$b_0 = \bar{y} - b_1 \bar{x} \tag{84}$$

9.4 PREDICTION INTERVALS

To predict y when $x = 5$ and show the 95% prediction interval with regression model in `results`:

`predict(results, newdata=data.frame(x=5), int="pred")`

10 ANOVA

10.1 ONE WAY ANOVA

1. **`results=aov(depVarColName~indepVarColName, data=tableName)`** Run ANOVA with data in `TableName`, factor data in `indepVarColName` column, and response data in `depVarColName` column.
2. **`summary(results)`** Summarize results
3. **`boxplot(depVarColName~indepVarColName, data=tableName)`** Boxplot of levels for factor

$$F = \frac{MS(\text{treatment})}{MS(\text{error})}, \quad df_1 = k - 1, \, df_2 = N - k \tag{85}$$

To find required sample size and power see `power.anova.test(...)`

11 Loading and using external data and tables

11.1 LOADING EXCEL DATA

1. Export your table as a CSV file (comma seperated file) from Excel.
2. Import your table into `MyTable` in R using:
   `MyTable=read.csv(file.choose())`

11.2 LOADING AN .RDATA FILE

You can either double click on the `.RData` file or use the menu:

- Windows: *File→Load Workspace...*
- Mac: *Workspace→Load Workspace File...*

11.3 USING TABLES OF DATA

1. To see all the available variables type: **`ls()`**
2. To see what's inside a variable, type its name.
3. If the variable `tableName` is a table, you can also type **`names(tableName)`** to see the column names or type **`head(tableName)`** to see the first few rows of data.
4. To access a column of data type **`tableName$columnName`**

An example demonstrating how to get the women's height data and find the mean:

```
> ls() # See what variables are defined
[1] "women" "x"
> head(women) #Look at the first few entries
  height weight
1     58    115
2     59    117
3     60    120
> names(women) # Just get the column names
[1] "height" "weight"
> women$height # Display the height data
 [1] 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72
> mean(women$height)  # Find the mean of the heights
[1] 65
```