

MAT 167: STATISTICS

FINAL EXAM

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SPRING 2009

Name: _____

Computer / Seat Number: _____

No books, notes, or friends. **Show your work.** You may use the attached equation sheet, R, and a calculator. No other materials. If you choose to use R, write what you typed on the test. Using any other program or having any other documents open on the computer will constitute cheating.

You have until the end of class to finish the exam, manage your time wisely.

If something is unclear quietly come up and ask me.

If the question is legitimate I will inform the whole class.

Express all final answers to 3 significant digits. Probabilities should be given as a decimal number unless a percent is requested. Circle final answers, ambiguous or multiple answers will not be accepted. Show steps where appropriate.

The exam consists of 24 questions for a total of 71 points on 9 pages.

This Exam is being given under the guidelines of our institution's **Code of Academic Ethics**. You are expected to respect those guidelines.

Points Earned: _____ out of 71 total points

Exam Score: _____

1. The following is a partial list of statistical methods that we have discussed:
 1. mean
 2. median
 3. mode
 4. standard deviation
 5. z-score
 6. percentile
 7. coefficient of variation
 8. scatter plot
 9. histogram
 10. pareto chart
 11. box plot
 12. normal-quantile plot
 13. confidence interval for a mean
 14. confidence interval for difference in means
 15. confidence interval for a proportion
 16. confidence interval for difference in proportions
 17. one sample mean test
 18. two independent sample mean test
 19. match pair test
 20. one sample proportion test
 21. two sample proportion test
 22. test of homogeneity
 23. test of independence
 24. linear correlation coefficient & test
 25. regression
 26. 1-way ANOVA

For each situation below, which method is most applicable?

- (a) (1 point) A researcher would like to estimate the mean weight of javalina.
 - (b) (1 point) A researcher wants to determine if bear weights are normally distributed.
 - (c) (1 point) An education researcher wants to determine if the probability a student will graduate from middle school is effected by their economic status (poor, lower middle class, middle class, ...).
 - (d) (1 point) A farmer wants to determine if the mean crop yield is the same for eight different brands of fertilizer.
 - (e) (1 point) A fertility researcher wants to determine if a new drug can decrease the proportion of infertile mice. Twenty mice are randomly divided into two groups, a treatment group and a control group.
2. (1 point) What test is a many sample generalization of the two sample t -test?
 3. (1 point) If the mean, median, and mode for a data set are different, what can you conclude about the data's distribution?

4. (2 points) Under what conditions can we approximate a binomial distribution as a normal distribution?
5. (1 point) What percent of data lies within one standard deviations of the mean as stated by the Empirical Rule?
6. (1 point) Why is it important to use random sampling?
7. (1 point) A sampling distribution characterizes what type of error?
8. For the following statements, determine if the calculation requires the use of a **population distribution** or a **sampling distribution**.
 - (a) (1 point) Computing a confidence interval for a proportion.
 - (b) (1 point) Computing an interval that contain 95% of individual's weights.
9. (1 point) If the normal approximation to the binomial is valid, write what the following binomial probability statement is approximately equal to in terms of the normal distribution.
$$P_{\text{binom}}(x > 10) \approx$$
10. (1 point) For ANOVA, what is the distribution of the test statistic? (Give the specific name.)
11. (2 points) A hypothesis test was conducted for $H_0 : \mu = 5$ and $H_a : \mu > 5$. The test statistic is $t = 2.2$, $n = 15$. Find the p-value.
12. (2 points) A histogram is a useful tool that can quickly communicate many traits about a set of data. List 4 useful pieces of information that an observer can easily assess using a histogram.

13. Provide **short succinct** written answers to the following conceptual questions.

- (a) (1 point) Give an example of a categorical type of variable.
- (b) (1 point) Which of the following measures of variation is least susceptible to outliers:
standard deviation, inter-quartile range, range
- (c) (1 point) What percent of data is greater than Q_3 ?
- (d) (1 point) What does the standard deviation represent conceptually **in words**? (Be concise but don't simply state the equation in words verbatim.)
- (e) (1 point) Why would a SAT percentile be preferred over a raw SAT score for college admissions committees?

14. (2 points) Car tires must not deform or explode when inflated up to their maximum pressure rating. Before distributing the tires, they must be tested. To test the safety of tires, an inspector randomly samples 50 tires (without replacement) from a batch of 5,000 that have been manufactured. The inspector inflates each of the fifty tires until they explode or deform to make sure they meet the minimum safety requirements. If none of the sampled tires fails the test, the tires will be distributed to dealers. If the batch contains 15 defective tires that will explode if selected, what is the probability that the batch will be rejected?

15. (2 points) If a class consists of 20 males and 8 females, what is the probability of drawing 4 females without replacement?

16. (2 points) You would like to conduct a study to estimate (at the 95% confidence level) the proportion of households that own one or more encyclopedias. What sample size do you need to estimate the proportion with a margin of error of 2%.

17. The following questions regard hypothesis testing in general.

(a) (1 point) When we conduct a hypothesis test, we assume something is true and calculate the probability of observing the sample data under this assumption. What do we assume is true?

(b) (1 point) If you reject H_0 but H_0 is true, what type of error has occurred? (Type I or Type II)

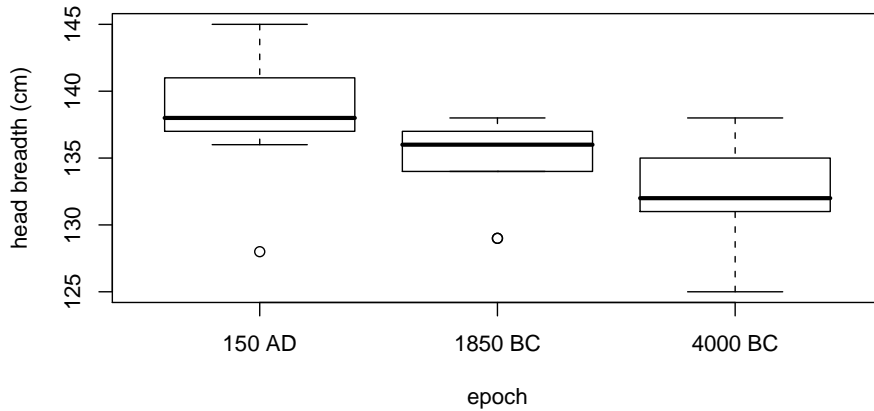
(c) (1 point) What variable represents the actual Type I error?

(d) (1 point) What does the power of a hypothesis test represent?

18. Eighteen students were randomly selected to take the SAT after having either no breakfast or a complete breakfast. A researcher would like to test the claim that students who eat breakfast score higher than students who do not.

Group without breakfast: SAT Score	480	510	530	540	550	560	600	620	660
Group with breakfast: SAT Score	460	500	530	520	580	580	560	640	690

- (a) (1 point) What type of hypothesis test will you use?
 - (b) (2 points) What are the test's requirements?
 - (c) (2 points) What are the hypothesis H_0 and H_a ?
 - (d) (1 point) What α will you use?
 - (e) (2 points) Conduct the hypothesis test. What is the p -value?
 - (f) (1 point) What is your formal decision?
 - (g) (2 points) State your final conclusion in words.
19. Samples of head breadths were obtained by measuring skulls of Egyptian males from three different epochs, and the measurements are listed below (based on data from *Ancient Races of the Tebaid*, by Thomas and Randall-Maciver). Changes in head shape over time suggest that interbreeding occurred with immigrant populations. Test the claim that the different epochs do not all have the same mean head breadth.
- A box plot of the data is shown below.



- (a) (1 point) What type of hypothesis test (of those discussed in class) should you use?
- (b) (1 point) What is the alternative hypothesis for this test?
- (c) (1 point) What alpha will you use?
- (d) (1 point) What is the response variable for this study?
- (e) (1 point) What is the factor variable for this study?
- (f) (1 point) The analysis of the data was run and the output is shown below: What is your

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
epoch	2	138.74	69.37	4.05	0.0305
Residuals	24	411.11	17.13		

final conclusion (not the formal decision)?

- (g) (1 point) Assuming the researcher rejected the null hypothesis, what is the probability of a Type I error for this study?

20. The following table lists the the fuel consumption (in miles/gallon) and weight (in lbs) of a vehicle.

Weight	3180	3450	3225	3985	2440	2500	2290
MPG	27	29	27	24	37	34	37

- (a) (2 points) Upon looking at the scatter plot of the data, the relationship of fuel consumption and milage looks linear. Is the linear relationship statistically significant? (**Justify your answer with an analysis.**)
- (b) (1 point) What percent of a vehicle's fuel consumption can be explained by its weight?
- (c) (2 points) You are designing a new vehicle and would like to be able to predict its fuel consumption. Write the equation for fitted model (with the actual values of the coefficients).
- (d) (1 point) What range of vehicle weights is the model valid for making predictions of fuel efficiency?
- (e) (1 point) What is the best predicted fuel consumption for a new vehicle that weights 3200 lbs?
- (f) (1 point) If the liner relationship had not been statistically significant, what is the best predicted fuel consumption for a new vehicle that weights 3200 lbs?

21. (2 points) A researcher is trying to determine the ideal temperature to brew coffee. A random sample of 8 of the top 100 coffee shops in New York City had their brewer temperatures (in Celsius) measured. The data is shown below.

88.6, 91.2, 87.5, 90.5, 85.4, 90.6, 93.5, 94.6

Construct a 90% confidence interval for the true population mean temperature using the above data. (**Assume σ is unknown.**)

22. (2 points) A ski resort is designing a new super tram to carry 40 people. If the mean weight of humans is approximately 165 lbs with a standard deviation of 25 lbs, what should the tram's maximum weight limit be so that it can carry the desired capacity 95% of the time?

23. Engineers must consider the breadths of male heads when designing motorcycle helmets. Men have head breadths that are normally distributed with a mean of 6.0 in and a standard deviation of 1.0 in (based on anthropometric survey data from Gordon, Churchill, et al.).

(a) (2 points) If 1 man is randomly selected, find the probability that his head breadth is greater than 6.1 in.

(b) (2 points) If 100 men are randomly selected, find the probability that their mean head breadth is greater than 6.1 in.

24. (2 points) Given $y = \{a, -2a, 4a\}$, where a is a constant, completely simplify the following expression:

$$\left(\sum y_i\right)^2 - 2$$

End of exam. Reference sheets follow.

Statistics Quick Reference

Card & R Commands

by Anthony Tanbakuchi, Version 1.8.2
<http://www.tanbakuchi.com>
 ANTHONY@TANBAKUCHI.COM
 Get R at: <http://www.r-project.org>
 R commands: bold typewriter text

1 Misc R

To make a vector v store data: $x=c(x1, x2, \dots)$
 Help: general `RSiteSearch("Search Phrase")`
 Get: function `?functionName`
 Get column of data from table:
`tableName$columnName`
 List all variables: `ls()`
 Delete all variables: `rm(list=ls())`

$$\sqrt{x} = \text{sqrt}(x) \quad (1)$$

$$x^n = x^n \quad (2)$$

$$n = \text{length}(x) \quad (3)$$

$$T = \text{table}(x) \quad (4)$$

2 Descriptive Statistics

2.1 NUMERICAL

Let $x=c(x1, x2, x3, \dots)$

$$\text{total} = \sum_{i=1}^n x_i = \text{sum}(x) \quad (5)$$

$$\text{min} = \text{min}(x) \quad (6)$$

$$\text{max} = \text{max}(x) \quad (7)$$

$$\text{six number summary} = \text{summary}(x) \quad (8)$$

$$\mu = \frac{\sum x_i}{N} = \text{mean}(x) \quad (9)$$

$$\bar{x} = \frac{\sum x_i}{n} = \text{mean}(x) \quad (10)$$

$$\bar{x} = P_{50} = \text{median}(x) \quad (11)$$

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \quad (12)$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \text{sd}(x) \quad (13)$$

$$CV = \frac{\sigma}{\mu} = \frac{s}{\bar{x}} \quad (14)$$

2.2 RELATIVE STANDING

$$z = \frac{x - \mu}{\sigma} = \frac{x - \bar{x}}{s} \quad (15)$$

Percentiles:

$$P_k = x_k, \text{ (sorted } x) \quad (16)$$

$$k = \frac{i-0.5}{n} \cdot 100\%$$

To find x_i given P_k , i is:

- $L = (k/100)n$
- if L is an integer: $i = L + 0.5$; otherwise $i = L$ and round up.

2.3 VISUAL

All plots have optional arguments:

- `main=""` sets title
- `xlab=""`, `ylab=""` sets x/y-axis label
- `type="p"` for point plot
- `type="l"` for line plot
- `type="b"` for both points and lines

Ex: plot(x, y, type="b", main="My Plot")

Plot Types:

`hist(x)` histogram
`stem(x)` stem & leaf
`boxplot(x)` box plot
`plot(T)` bar plot, `T=table(x)`
`plot(x, y)` scatter plot, x, y are ordered vectors
`plot(t, y)` time series plot, t, y are ordered vectors
`curve(expr, xmin, xmax)` plot expr involving x

2.4 ASSESSING NORMALITY

Q-Q plot: `qqnorm(x)`; `qqline(x)`

3 Probability

Number of successes x with n possible outcomes. (Don't double count!)

$$P(A) = \frac{x}{n} \quad (17)$$

$$P(\bar{A}) = 1 - P(A) \quad (18)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \quad (19)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ if } A, B \text{ mut. excl.} \quad (20)$$

$$P(A \text{ and } B) = P(A) \cdot P(B|A) \quad (21)$$

$$P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A, B \text{ independent} \quad (22)$$

$$n! = n(n-1) \cdots 1 = \text{factorial}(n) \quad (23)$$

$${}_n P_k = \frac{n!}{(n-k)!} \text{ Perm. no elem. alike} \quad (24)$$

$${}_n A_k = \frac{n!}{n_1! n_2! \cdots n_k!} \text{ Perm. } n_1 \text{ alike, } \dots \quad (25)$$

$${}_n C_k = \frac{n!}{(n-k)! k!} = \text{choose}(n, k) \quad (26)$$

4 Discrete Random Variables

$$P(x_i) : \text{probability distribution} \quad (27)$$

$$E = \mu = \sum x_i \cdot P(x_i) \quad (28)$$

$$\sigma = \sqrt{\sum (x_i - \mu)^2 \cdot P(x_i)} \quad (29)$$

4.1 BINOMIAL DISTRIBUTION

$$\mu = n \cdot p \quad (30)$$

$$\sigma = \sqrt{n \cdot p \cdot q} \quad (31)$$

$$P(x) = {}_n C_x p^x q^{n-x} = \text{dbinom}(x, n, p) \quad (32)$$

4.2 POISSON DISTRIBUTION

$$P(x) = \frac{\mu^x \cdot e^{-\mu}}{x!} = \text{dpois}(x, \mu) \quad (33)$$

5 Continuous random variables

CDF $F(x)$ gives area to the left of x , $F^{-1}(p)$ expects p is area to the left.

$$f(x) : \text{probability density} \quad (34)$$

$$E = \mu = \int_{-\infty}^{\infty} x \cdot f(x) dx \quad (35)$$

$$\sigma = \sqrt{\int_{-\infty}^{\infty} (x - \mu)^2 \cdot f(x) dx} \quad (36)$$

$$F(x) : \text{cumulative prob. density (CDF)} \quad (37)$$

$$F^{-1}(x) : \text{inv. cumulative prob. density} \quad (38)$$

$$f(x) = \frac{d}{dx} F(x) \quad (39)$$

$$p = P(x < x') = F(x') \quad (40)$$

$$x' = F^{-1}(p) \quad (41)$$

$$p = P(x > a) = 1 - F(a) \quad (42)$$

$$p = P(a < x < b) = F(b) - F(a) \quad (43)$$

5.1 UNIFORM DISTRIBUTION

$$p = P(a < u') = F(u') \quad (44)$$

$$u' = F^{-1}(p) = \text{qunif}(p, \text{min}=a, \text{max}=b) \quad (45)$$

5.2 NORMAL DISTRIBUTION

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2} \frac{(x-\mu)^2}{\sigma^2}} \quad (46)$$

$$p = P(z < z') = F(z') = \text{pnorm}(z') \quad (47)$$

$$z' = F^{-1}(p) = \text{qnorm}(p) \quad (48)$$

$$p = P(x < x') = F(x') \quad (49)$$

$$x' = \text{pnorm}(x', \text{mean}=\mu, \text{sd}=\sigma) \quad (49)$$

$$x' = F^{-1}(p) \quad (50)$$

$$= \text{qnorm}(p, \text{mean}=\mu, \text{sd}=\sigma) \quad (50)$$

5.3 t-DISTRIBUTION

$$p = P(t < t') = F(t') = \text{pt}(t', \text{df}) \quad (51)$$

$$t' = F^{-1}(p) = \text{qt}(p, \text{df}) \quad (52)$$

5.4 χ^2 -DISTRIBUTION

$$p = P(\chi^2 < \chi'^2) = F(\chi'^2) \quad (53)$$

$$= \text{pchisq}(\chi'^2, \text{df}) \quad (53)$$

$$\chi'^2 = F^{-1}(p) = \text{qchisq}(p, \text{df}) \quad (54)$$

5.5 F-DISTRIBUTION

$$p = P(F < F') = F(F') \quad (55)$$

$$= \text{pf}(F', \text{df1}, \text{df2}) \quad (55)$$

$$F' = F^{-1}(p) = \text{qf}(p, \text{df1}, \text{df2}) \quad (56)$$

6 Sampling distributions

$$\mu_{\bar{x}} = \mu \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \quad (57)$$

$$\mu_{\bar{p}} = p \quad \sigma_{\bar{p}} = \sqrt{\frac{pq}{n}} \quad (58)$$

7 Estimation

7.1 CONFIDENCE INTERVALS

$$\text{proportion: } \hat{p} \pm E, \quad E = z_{\alpha/2} \cdot \sigma_{\bar{p}} \quad (59)$$

$$\text{mean } (\sigma \text{ known): } \bar{x} \pm E, \quad E = z_{\alpha/2} \cdot \sigma_{\bar{x}} \quad (60)$$

$$\text{mean } (\sigma \text{ unknown, use } s): \bar{x} \pm E, \quad E = t_{\alpha/2} \cdot \sigma_{\bar{x}} \quad (61)$$

$$df = n - 1$$

$$\text{variance: } \frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L} \quad (62)$$

$$df = n - 1$$

$$2 \text{ proportions: } \Delta \hat{p} \pm z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} \quad (63)$$

$$2 \text{ means (indep): } \Delta \bar{x} \pm t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (64)$$

$$df \approx \text{min}(n_1 - 1, n_2 - 1)$$

$$\text{matched pairs: } \bar{d} \pm t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}, \quad d_i = x_i - y_i \quad (65)$$

$$df = n - 1$$

7.2 CI CRITICAL VALUES (TWO SIDED)

$$z_{\alpha/2} = F_z^{-1}(1 - \alpha/2) = \text{qnorm}(1 - \alpha/2) \quad (66)$$

$$t_{\alpha/2} = F_t^{-1}(1 - \alpha/2) = \text{qt}(1 - \alpha/2, \text{df}) \quad (67)$$

$$\chi^2_L = F_{\chi^2}^{-1}(\alpha/2) = \text{qchisq}(\alpha/2, \text{df}) \quad (68)$$

$$\chi^2_R = F_{\chi^2}^{-1}(1 - \alpha/2) = \text{qchisq}(1 - \alpha/2, \text{df}) \quad (69)$$

7.3 REQUIRED SAMPLE SIZE

$$\text{proportion: } n = \hat{p} \hat{q} \left(\frac{z_{\alpha/2}}{E} \right)^2 \quad (70)$$

$$(\hat{p} = \hat{q} = 0.5 \text{ if unknown})$$

$$\text{mean: } n = \left(\frac{z_{\alpha/2} \cdot \hat{\sigma}}{E} \right)^2 \quad (71)$$

8 Hypothesis Tests

Test statistic and R function (when available) are listed for each.

Optional arguments for **hypothesis tests**:

alternative="two.sided" can be:
"two.sided", "less", "greater"

conf.level=0.95 constructs a 95% confidence interval. Standard CI only when **alternative**="two.sided".

Optional arguments for **power calculations & Type II error**:

alternative="two.sided" can be:
"two.sided" or "one.sided"

sig.level=0.05 sets the significance level α .

8.1 1-SAMPLE PROPORTION

$H_0: p = p_0$

prop.test(x, n, p=p0, alternative="two.sided")

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0q_0/n}} \quad (72)$$

8.2 1-SAMPLE MEAN (σ KNOWN)

$H_0: \mu = \mu_0$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \quad (73)$$

8.3 1-SAMPLE MEAN (σ UNKNOWN)

$H_0: \mu = \mu_0$

t.test(x, mu= μ_0 , alternative="two.sided")

Where **x** is a vector of sample data.

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}, \quad df = n - 1 \quad (74)$$

Required Sample size:

power.t.test(delta= h , sd= σ , sig.level= α , power=1 - β , type="one.sample", alternative="two.sided")

8.4 2-SAMPLE PROPORTION TEST

$H_0: p_1 = p_2$ or equivalently $H_0: \Delta p = 0$

prop.test(x, n, alternative="two.sided")

where: **x**=c(x_1, x_2) and **n**=c(n_1, n_2)

$$z = \frac{\Delta \hat{p} - \Delta p_0}{\sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}}, \quad \Delta \hat{p} = \hat{p}_1 - \hat{p}_2 \quad (75)$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2}, \quad \hat{q} = 1 - \hat{p} \quad (76)$$

Required Sample size:

power.prop.test(p1= p_1 , p2= p_2 , power=1 - β , sig.level= α , alternative="two.sided")

8.5 2-SAMPLE MEAN TEST

$H_0: \mu_1 = \mu_2$ or equivalently $H_0: \Delta \mu = 0$

t.test(x1, x2, alternative="two.sided")

where: **x1** and **x2** are vectors of sample 1 and sample 2 data.

$$t = \frac{\Delta \bar{x} - \Delta \mu_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}, \quad df \approx \min(n_1 - 1, n_2 - 1), \quad \Delta \bar{x} = \bar{x}_1 - \bar{x}_2 \quad (77)$$

Required Sample size:

power.t.test(delta= h , sd= σ , sig.level= α , power=1 - β , type="two.sample", alternative="two.sided")

8.6 2-SAMPLE MATCHED PAIRS TEST

$H_0: \mu_d = 0$

t.test(x, y, paired=TRUE, alternative="two.sided")

where: **x** and **y** are ordered vectors of sample 1 and sample 2 data.

$$t = \frac{\bar{d} - \mu_{d0}}{s_d/\sqrt{n}}, \quad d_i = x_i - y_i, \quad df = n - 1 \quad (78)$$

Required Sample size:

power.t.test(delta= h , sd= σ , sig.level= α , power=1 - β , type="paired", alternative="two.sided")

8.7 TEST OF HOMOGENEITY, TEST OF INDEPENDENCE

$H_0: p_1 = p_2 = \dots = p_m$ (homogeneity)

$H_0: X$ and Y are independent (independence)

chisq.test(D)

Enter table: **D**=data.frame(**c1, c2, ...**), where **c1, c2, ...** are column data vectors.

Or generate table: **D**=table(**x1, x2**), where **x1, x2** are ordered vectors of raw categorical data.

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}, \quad df = (\text{num rows} - 1)(\text{num cols} - 1) \quad (79)$$

$$E_i = \frac{(\text{row total})(\text{column total})}{(\text{grand total})} = np_i \quad (80)$$

For 2×2 contingency tables, you can use the Fisher Exact Test:

fisher.test(D, alternative="greater")

(must specify alternative as greater)

9 Linear Regression

9.1 LINEAR CORRELATION

$H_0: \rho = 0$

cor.test(x, y)

where: **x** and **y** are ordered vectors.

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y}, \quad t = \frac{r - 0}{\sqrt{\frac{1-r^2}{n-2}}} \quad df = n - 2 \quad (81)$$

9.2 MODELS IN R

MODEL TYPE	EQUATION	R MODEL
linear 1 indep var	$y = b_0 + b_1 x_1$	$y \sim x_1$
... 0 intercept	$y = 0 + b_1 x_1$	$y \sim 0 + x_1$
linear 2 indep vars	$y = b_0 + b_1 x_1 + b_2 x_2$	$y \sim x_1 + x_2$
... interaction	$y = b_0 + b_1 x_1 + b_2 x_2 + b_{12} x_1 x_2$	$y \sim x_1 + x_2 + x_1 * x_2$
polynomial	$y = b_0 + b_1 x_1 + b_2 x_1^2$	$y \sim x_1 + 1(x_1^2)$

9.3 REGRESSION

Simple linear regression steps:

1. Make sure there is a significant linear correlation.
2. **results=lm(y~x)** Linear regression of y on x vectors
3. **results** View the results
4. **plot(x, y); abline(results)** Plot regression line on data
5. **plot(x, results\$residuals)** Plot residuals

$$y = b_0 + b_1 x_1 \quad (82)$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad (83)$$

$$b_0 = \bar{y} - b_1 \bar{x} \quad (84)$$

9.4 PREDICTION INTERVALS

To predict y when $x = 5$ and show the 95% prediction interval with regression model in results:

predict(results, newdata=data.frame(x=5), int="pred")

10 ANOVA

10.1 ONE WAY ANOVA

1. **results=aov(depVarColName~indepVarColName, data=tableName)** Run ANOVA with data in **tableName**, factor data in **indepVarColName** column, and response data in **depVarColName** column.
2. **summary(results)** Summarize results
3. **boxplot(depVarColName~indepVarColName, data=tableName)** Boxplot of levels for factor

$$F = \frac{MS(\text{treatment})}{MS(\text{error})}, \quad df_1 = k - 1, df_2 = N - k \quad (85)$$

To find required sample size and power see **power.anova.test(...)**

11 Loading and using external data and tables

11.1 LOADING EXCEL DATA

1. Export your table as a CSV file (comma separated file) from Excel.
2. Import your table into R using:
MyTable=read.csv(file.choose())

11.2 LOADING AN .RDATA FILE

You can either double click on the .RData file or use the menu:

- Windows: **File→Load Workspace...**
- Mac: **Workspace→Load Workspace File...**

11.3 USING TABLES OF DATA

1. To see all the available variables type: **ls()**
2. To see what's inside a variable, type its name.
3. If the variable **tableName** is a table, you can also type **names(tableName)** to see the column names or type **head(tableName)** to see the first few rows of data.
4. To access a column of data type **tableName\$columnName**

An example demonstrating how to get the women's height data and find the mean:

```
> ls() # See what variables are defined
[1] "women" "x"
> head(women) # Look at the first few entries
  height weight
1    58    115
2    59    117
3    60    120
> names(women) # Just get the column names
[1] "height" "weight"
> women$height # Display the height data
[1] 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72
> mean(women$height) # Find the mean of the heights
[1] 65
```